淡江大學98學年度碩士班招生考試試題

系別：數學學系　　科目：高 等 微 積 分

准帶項目請打「V」
簡單型計算機

本試題共 | 頁，7 大題

1. A set S in R is said to be compact if, and only if, every open covering of S contains a finite subcover. Show that $(0,1)$ is not compact. (10%)

2. Let $\{a_n\}$ be a sequence in R. If $\lim_{n\to\infty} a_n$ exists, show that $\{a_n\}$ is bounded. (15%)

3. Show that $f(x)=|x|$ is continuous but not differentiable at $x=0$. (15%)

4. Let $\{f_n\}$ be a sequence of functions on $D\subseteq R$ to R and f be function on D. We say $\{f_n\}$ converges uniformly to f on D if for every $\varepsilon>0$, there is N such that for all $n\geq N$ implies $|f_n(x)-f(x)|<\varepsilon$ for all $x\in D$. (15%)

   (a) Show that $f_n(x)=\frac{1}{n}\sin(nx)$ converges uniformly to 0 on $D=R$.

   (b) Show that $f_n(x)=x^n$ doesn't converges uniformly on $D=[0,1]\subset R$.

5. (a) Let f be a continuous defined on $[a,b]$ into R. Show that (15%)

$$\frac{d}{dx}\int_a^x f(t)dt=f(x).$$

   (b) Evalulate $\frac{d}{dx}\int_1^{x^2} e^{\sqrt{t}}\sin(2t)dt$.

6. Let $f(x,y)=\begin{cases}\frac{xy^2}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$ (15%)

   (a) Show that $f(x,y)$ is continuous at $(0,0)$.

   (b) Find $(D_1f)(x,y)$ for all $x,y\in R^2$.

   (c) Find $(D_2f)(x,y)$ for all $x,y\in R^2$.

7. (a) Is it possible to solve (15%)

$$uy+vx+w+x^2=0$$
$$uvw+x+y+1=0$$

   for (x,y) in terms of (u,v,w) near $(u,v,w)=(2,1,0)$, $(x,y)=(-1,0)$.

   (b) Find $\frac{\partial x}{\partial u}$, $\frac{\partial y}{\partial w}$ at $(u,v,w)=(2,1,0)$.