淡江大學 97 學年度碩士班招生考試試題

系別：數學學系　　　　科目：高 等 微 積 分

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(20%) 1. (a) Give a definition of compactness

(b) Give a definition of uniform continuity.

(c) Let f be a continuous function on a bounded closed interval $[a, b]$ into R. Show that f is uniformly continuous on $[a, b]$.

(10%) 2. If $s_1 = \sqrt{2}$, and $s_{n+1} = \sqrt{2+\sqrt{s_n}}$, $n = 1, 2, 3, \ldots$, prove that $\lim_{n\to\infty} s_n$ exists.

(10%) 3. Let f be defined on $[a, b]$. If f is differentiable at $c \in (a,b)$, show that f is continuous at c.

(20%) 4. (a) Let α be a monotonically increasing function on $[a, b]$ and f be a bounded function on $[a, b]$. Give a definition of the Riemann-Stieltjes integral of f with respect to α, over $[a, b]$.

(b) Evaluate

$$\int_0^2 e^x d[x]$$

where $[x]$ is Gauss Integer function.

(20%) 5. (a) Give an example to show that f_n is Riemann integrable on $[a, b]$ such that $f(x) = \lim_{n\to\infty} f_n(x)$, but

$$\lim_{n\to\infty} \int_a^b f_n(x)dx \neq \int_a^b f(x)dx$$

(b) Let f_n is Riemann integrable over $[a, b]$. Suppose f_n converges to f uniformly. Show that f is Riemann integrable over $[a, b]$ and

$$\lim_{n\to\infty} \int_a^b f_n dx = \int_a^b f dx$$

(10%) 6. (a) Is it possible to solve

$$xy^2 + xzu + yv^2 = 3$$
$$u^3yz + 2xv - u^2v^2 = 2$$

for u, v interms of x, y, z near $(x,y,z) = (1,1,1)$, $(u,v) = (1,1)$?

(b) Find $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$ at (1,1,1).

(10%) 7. Let $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

(a) Is $f(x,y)$ continuous at (0,0)? Explain why.

(b) Find $(D_1 f)(x,y)$ and $(D_2 f)(x,y)$ for all $x, y \in R^2$.