

系列：數學學系

科目：高等微積分

准帶項目請打「V」

λ

簡單型計算機

本試題共

頁

1. Let $f(x) = \begin{cases} x \cos^2(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0, \end{cases}$ (1090)
- (a) Is f continuous at 0? Prove it.
- (b) Is f differentiable at 0? Prove it.
2. Does $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^6 + y^2}$ exist? Prove it. (1090)
3. Use definition to evaluate $\int_0^1 x dx^2$. (1090)
4. Let $f(x) = 2 \sin x$. Prove or disprove that f is uniformly continuous. (1090)
5. Suppose each f_n is continuous and $f_n \rightarrow f$ uniformly. Prove or disprove that f is continuous. (1090)
6. (a) $\frac{d}{dx} \int_1^{x^2} e^{\sin t} \ln(t^2+1) dt = ?$ (b) $\frac{\partial}{\partial y} [e^x \ln(y^2) \sin y] = ?$ (1090)
7. Suppose $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ ($m, n \geq 2$). Define $f'(x)$. (1090)
8. Suppose $f_1: \mathbb{R}^5 \rightarrow \mathbb{R}^1$ is given by $f_1(x_1, x_2, y_1, y_2, y_3) = 2e^{x_1} + x_2 y_1 - 4y_2 + 3$,
 $f_2: \mathbb{R}^5 \rightarrow \mathbb{R}^1$ is given by $f_2(x_1, x_2, y_1, y_2, y_3) = x_2 \cos x_1 - 6x_1 + 2y_1 - y_3$,
 and $f = (f_1, f_2): \mathbb{R}^5 \rightarrow \mathbb{R}^2$. If $a = (0, 1)$, $b = (3, 2, 7)$, then $f(a, b) = 0$.
- (a) What is $f'(a, b)$? (2090)
- (b) Does there exist a function that is defined in a neighborhood of b and maps b into a ? Explain
9. Suppose $f, g: \mathbb{R}^1 \rightarrow \mathbb{R}^1$ are continuous functions. Prove that $(g \circ f)(x) = g(f(x))$ is also continuous. (1070)