淡江大學九十四學年度碩士班招生考試試題 21-1

系別：數學學系　　　　科目：高 等 微 積 分

准帶項目請打「∨」	
X	簡單型計算機

本試題共　|　頁

#R is the set of real numbers

1. Let A and B be nonempty bounded subsets of R.
   (a) Write the definition of SupA (the least upper bound of A).(3 points)
   (b) Let A+B ={x+y : x∈A and y∈B}.
       Prove that Sup(A+B)≤SupA + SupB. (7 points)

2. Let f : R → R be differentiable and a, b∈ R, a < b, $f'(a) < 0$.
   (a) Explain the terminology: f is differentiable at a. (3 points)
   (b) Show that f(a) is not a minimum of f on [a,b]. (7 points)

3. Let g be dined by g(x)=$x^2 \sin(\frac{1}{x})$ if x ≠ 0 and g(0)=0.
   (a) Prove that g is differentiable at 0 and that $g'(0)=0$. (5 points)
   (b) Show that $g'(x)$ is not continuous at 0 .(10 points)

4. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point(1,-1) if x^2-xy+y^2=3 .(11 points)

5. Let g be dined by g(x,y)= $\frac{xy(x^2-y^2)}{x^2+y^2}$ if (x,y)≠(0,0) and g(0,0)= 0.
   Find $\frac{\partial^2 g}{\partial x \partial y}(0,0)$ and $\frac{\partial^2 g}{\partial y \partial x}(0,0)$. (10 points)

6. Show that $\sum_{n=1}^{\infty} \frac{\cos(nx)}{n(n+1)}$ converges uniformly on R . (10 points)

7. Evaluate each of the following : (Each has 7 points)
   (a) $\sum_{n=0}^{\infty} \frac{(\frac{1}{2})^n}{(n+1)}$.　　(b) $\int_0^1 (\int_x^1 \sin(y^2) dy) dx$.

8. Let u(x,y)=x^2-y^2 and v(x,y)=2xy . Compute $\frac{\partial x}{\partial u}$ in term of x and y. (10 points)

9. Let f : R → R be a continuous function.　Prove that the set
   {(x, f(x))| x∈ R} is a closed set in the plane. (10 points)