

## 淡江大學九十三年學年度碩士班招生考試試題

系別：數學學系

科目：高等微積分

准帶項目請打「○」否則打「×」

簡單型計算機

X

本試題共 1 頁

1. Suppose  $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ . Does  $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$  exist? Explain!
2. Suppose  $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) \cos(\frac{2}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ . Is  $f$  differentiable at 0? Explain!
3. Define  $\int_a^b f(x) dx$  and use the definition to evaluate  $\int_0^1 x dx$ .
4. (a) Find  $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$  (b) If  $y^2 = x^2 + \sin(xy)$ , find  $\frac{dy}{dx}$
5. Find the interval of convergence of  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$
6. Suppose  $f$  is continuous on  $[a,b]$  and  $F(x) = \int_a^x f(t) dt$ . Prove that  $F'(x) = f(x)$
7. Suppose each  $f_n$  is Riemann integrable and  $f_n \rightarrow f$  uniformly on  $[a,b]$ . Prove that  $f$  is integrable and  $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \int_a^b f_n(x) dx$
8. State the Inverse Function Theorem.
9. State the Implicit Function Theorem.
10. Let  $f(x) = \frac{1}{x} \forall x \in (0,1)$ , is  $f$  uniformly continuous on  $(0,1)$ ? Explain.