

淡江大學九十二學年度碩士班招生考試試題

系別：數學學系

科目：高等微積分

准帶項目請打「○」否則打「×」
簡單型計算機
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本試題共 1 頁

- Let A be a subset of \mathbb{R} (the set of real numbers) and $F: \mathbb{R} \rightarrow \mathbb{R}$. State or explain the following theorem or terminology.
 - A is dense in \mathbb{R} . (3 points)
 - F is uniformly continuous on \mathbb{R} . (3 points)
 - Intermediate Value Theorem. (3 points)
 - Cauchy sequence in \mathbb{R} . (3 points)
 - Archimedean Principle. (3 points)
- Find $\frac{dy}{dx}$ if $y = \int_0^{\cos x} \sqrt{1+t^2} dt$. (10 points)
- Find a function $f: [0,1] \rightarrow \mathbb{R}$ such that f is not Riemann integrable on $[0,1]$ but $|f|$ is Riemann integrable on $[0,1]$. (10 points)
- Let $a_n > 0$ for all natural numbers n . Show that if $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} a_n^2$ converges. (10 points)
- Let $f_n(x) = n(1-x)x^n$, $0 \leq x \leq 1$.
 - Find the limit function $f(x)$ of $f_n(x)$. (5 points)
 - Show that $f_n(x)$ is not uniformly convergent on $[0,1]$. (10 points)
- Evaluate each of the following:
 - $\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)(n+3)}$. (7 points)
 - $\int_0^1 dx \int_{2x}^2 e^{y^2} dy$. (7 points)
 - $\lim_{n \rightarrow \infty} (1 + \frac{x}{n})^n$. (6 points)
- Let $F: \mathbb{R} \rightarrow \mathbb{R}$ be continuous. Show that if $F(x) = 0$ for every rational number x , then $F(x) = 0$ for every real number x . (10 points)
- Let E be a nonempty subset of \mathbb{R} . For x in \mathbb{R} , let $d(x) = \inf\{|x-y|: y \in E\}$.
 - Show that $|d(x) - d(y)| \leq |x-y|$. (5 points)
 - Show that if E is closed and $x \notin E$, then $d(x) > 0$. (5 points)