淡江大學九十一學年度碩士班招生考試試題

系別：數學系　　　　科目：高等微積分

准帶項目請打「○」否則打「X」	
計算機	字典
X	X

33-1

一、(15%)

(1) Find $f'(x)$ if $f(x) = x^x$, $x > 0$.

(2) Find $\dfrac{dy}{dx}$, if $yx^2 = \cos x + e^y \ln(1+x^2)$.

(3) If $f(x,y)=\begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} , & (x,y)\neq(0,0) \\ 0 , & (x,y)=(0,0), \end{cases}$

find $\dfrac{\partial}{\partial x}\dfrac{\partial}{\partial y} f(x,y)\Big|_{(x,y)=(0,0)}$.

二、Evaluate each of the following integrals (15%) :

(1) $\int_1^2 (\ln x)^3 dx$.

(2) $\int \dfrac{dx}{x^2-1}$.

(3) $\int_a^{\frac{a+b}{2}} (x-a)(x-\dfrac{2a+b}{3})dx$.

三、(32%)

(1) If f is continuous at $c\in(a , b)$ and $f(c)=\dfrac{1}{2}$, show that there is a $\delta>0$ such that

$$f(x) > \frac{1}{4} \quad \forall x\in(c-\delta, c+\delta)\subseteq(a,b) .$$

(2) If f is nonnegative and continuous on [0, 1] and $f(\frac{1}{2})=\frac{1}{2}$,

show that $\int_0^1 f(x)dx > 0$.

(3) If $f(x)=x^2$, show that f is uniformly continuous on [0 , 1] but is not uniformly continuous on $[0,\infty)$.

(4) If f is continuous on [0, 1] , show that f is bounded on [0 , 1] .

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計算機	字典
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四、Evaluate each of the following series (10%):

(1) $\sum_{n=1}^{\infty} \frac{n^2}{n!}$, (2) $\sum_{n=0}^{\infty} r^n$, where r is real number.

五、If f is a bounded function on $[a, b]$, and $P=\{a=x_0, x_1, x_2 \ldots, x_n=b\}$ is a partition of $[a, b]$. Let $M_i=\sup\{f(x): x\in[x_{i-1}, x_i]\}$,

$m_i=\inf\{f(x): x\in[x_{i-1}, x_i]\}$, $\Delta x_i = x_i - x_{i-1}$, $i = 1,2,\ldots,n$,

$$U(P,f)=\sum_{i=1}^{n} M_i \Delta x_i \quad , \quad L(P,f)=\sum_{i=1}^{n} m_i \Delta x_i ,$$

$\underline{\mathrm{I}}=\sup\{ L(P,f) : P$ is a partition of $[a, b]\}$,

$\overline{\mathrm{I}}=\inf\{ U(P,f) : P$ is a partition of $[a, b]\}$.

We say that f is Riemann integral on $[a, b]$ if $\underline{\mathrm{I}}=\overline{\mathrm{I}}$, in this case, we write $\int_a^b f(x)dx = \underline{\mathrm{I}} = \overline{\mathrm{I}}$.

(1) If $Q = P\cup\{c\}$ where $c\in(a,b)$, $c\notin P$, show that

$$U(Q,f)\le U(P,f) ,$$

$$L(Q,f)\ge L(P,f) .$$ (8%)

(2) If P,Q are any partition of $[a, b]$, show that

$$L(P,f)\le U(Q,f) .$$ (5%)

(3) Given $\varepsilon>0$, if there is a partition P_ε of $[a, b]$ such that

$$U(P_\varepsilon, f) - L(P_\varepsilon, f) \le \varepsilon ,$$

show that f is Riemann integrable on $[a, b]$. (5%)

(4) If f is continuous on $[a, b]$, show that f is Riemann integrable on $[a, b]$. (10%)