淡江大學九十學年度碩士班招生考試試題

系別：數學學系

科目：高等微積分

准帶項目請打「○」否則打「x」	
計算機	字典
x	x

本試題共 1. 頁

1. Let $f: \mathbb{R} \to \mathbb{R}$ be a function (12 points)
   (a) State the definition for f to be continuous.
   (b) State the definition for f to be uniformly continuous.
   (c) Give an example of a nonconstant uniformly continuous function.
   (d) Give an example of f which is continuous while it's not uniformly continuous.

2. (a) If $f: \mathbb{R} \to \mathbb{R}$ and f' exists on a neighborhood of $x = a$ and $\lim_{x \to a} f'(x) = L$. Show that $f'(a) = L$.
   (b) Let $g(x) = \begin{cases} 1 & x < 0 \\ 0 & x \geq 0 \end{cases}$, can g be the derivative of any function? (12 pts)

3. $\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}} = ?$ Justify your answer. (10 pts)

4. Use power series expansions to show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$, justify your steps. (10 pts)

5. $\int_0^\infty e^{-x^2} dx = ?$ Justify your answer. (8 pts)

6. Let $f_n: [0,1] \to \mathbb{R}$ be Riemann integrable, $f: [0,1] \to \mathbb{R}$
   If $f_n \to f$ uniformly, show that f is Riemann integrable and $\int_0^1 f_n(x)\,dx \to \int_0^1 f(x)\,dx$ as $n \to \infty$. (12 pts)

7. Let $f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$
   (a) Show that f is continuous at $(0,0)$.
   (b) Show that all directional derivatives at $(0,0)$ exists, while f is not differentiable at $(0,0)$ (12 pts)

8. Show that the system of equations (12 pts)
$$3x + y - z + u^2 = 0$$
$$x - y + 2z + u = 0$$
$$2x + 2y - 3z + 2u = 0$$
can be solved for x, y, u in terms of z; for x, z, u in terms of y; for y, z, u in terms of x; but not for x, y, z in terms of u.

9. Show that every open set in $\mathbb{R}$ is a countable disjoint union of open intervals. (12 pts)