淡江大學八十九學年度碩士班招生考試試題

系別：數學學系　　　　科目：高等微積分

本試題共 1 頁

1. Let f be continuous on $[a,b]$. Show that f is uniformly continuous. (10%)

2. Suppose f is continuous on $[a,b]$ and $F(x)=\int_a^x f(t)\,dt$. Prove or disprove that $F'(x)=f(x)$. (10%)

3. Let $f(x)=\begin{cases} 0 & \text{if } x<0 \\ 1 & \text{if } x\geq 1. \end{cases}$

   Can you find a function F such that $F''(x)=f(x)$? Give reasons. (10%)

4. Suppose $f(x,y)=\begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y)\neq(0,0) \\ 0 & \text{if } (x,y)=(0,0) \end{cases}$ (10%)

   (a) Find $D_u f(0,0)$ where $u=(u_1,u_2)$ is a unit vector

   (b) Is f differentiable at $(0,0)$? Explain.

5. Suppose (X,d) is a metric space and $T:(X,d)\to(X,d)$ is a mapping such that $d(Tx,Ty)\leq k\,d(x,y)$, where $0<k<1$. Prove that for each $x\in X$, the sequence $\{T^n x\}$ converges to the unique fixed point of T. (10%)

6. Suppose that $\lim_{n\to\infty} f_n(x)=f(x)$ for each $x\in[0,1]$. Prove or disprove that $\lim_{n\to\infty}\int_a^b f_n(x)\,dx=\int_a^b f(x)\,dx$. (10%)

7. (a) State the implicit function Theorem

   (b) State the inverse function Theorem. (10%)

8. Suppose $f:[0,1]\to[0,1]$ is a continuous function such that $f(\frac{1}{2})=\frac{1}{2}$. Prove or disprove that $\int_0^1 f(x)\,dx>0$. (10%)

9. Let $\sum a_n$ be a series of real numbers which converges, but <u>not</u> absolutely. Fix $\alpha\in\mathbb{R}$, prove or disprove that there exists a re-arrangement $\sum a_n'$ such that $\sum a_n'=\alpha$ (20%)