淡江大學八十八學年度碩士班招生考試試題

系別：數學學系　　　科目：高等微積分　　　本試題共 2 頁

1. (a) Given real numbers a and b, if $a \leqq b+\varepsilon$ for all $\varepsilon > 0$, show that $a \leqq b$.

   (b) If a is a nonnegative number and $a \leq \varepsilon$ for all $\varepsilon > 0$, show that $a = 0$. (12%)

2. If $\{a_n\}$ is a sequence of real numbers such that $\lim_{n\to\infty} a_n = l$ and $a_n \leq a_{n+1}$, $\forall n$, show that $a_n \leq l$, $\forall n$. (10%)

3. Let $f: R \to R$, $a \in R$. Suppose f is continuous at a and $f(a) > 0$. Show that $\exists\, \delta > 0$ such that $f(x) > 0$ $\forall x \in (a-\delta, a+\delta)$. (10%)

4. If $f: [a,b] \to [0,\infty)$ is continuous and $\int_a^b f(x)\,dx = 0$, show that $f(x) = 0$ $\forall x \in [a,b]$. (10%)

5. Show that $e^x \geq 1+x$ for all real number x. (8%)

6. A function f is said to satisfy a Lipschitz condition of order α on an interval (a,b), if there exists a positive number M such that

$$|f(x)-f(y)| \leq M\,|x-y|^{\alpha} \quad \text{for all } x, y \in (a,b).$$

If f satisfies a Lipschitz condition of order $\alpha > 1$ on (a,b), show that f must be a constant on (a,b). (10%)

◀注意背面尚有試題▶

試題雙面印製

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7. Prove that a contraction f of a complete metric space S has a unique fixed point. (14%)

8. If f is continuous on $[a,b]$, show that f is Riemann integrable on $[a,b]$. (12%)

9. If $\sum_{n=1}^{\infty} a_n$ is convergent and $\{b_n\}$ is a monotonic convergent sequence, show that $\sum_{n=1}^{\infty} a_n b_n$ is convergent. (14%)