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淡江大學八十七學年度碩士班入學考試試題

系別： 數學系 科目： 高等微積分

本試題共 / 頁

10 points for each problem.

1. Let f be continuoous on $[a, b]$. Show that f is uniformly continuous.

2. Does there exist a differentiable function f on $(-1, 1)$ such that its derivative is given by $g(x) = \begin{cases} -1, & -1 < x \leq 0 \\ 1, & 0 < x < 1 \end{cases}$
Explain why.

3. Suppose that $\{a_n\}$ is a monotone decresaing sequence of positive numbers. Show that if the series $\sum a_n$ converges, then $\lim(na_n) = 0$. Find a counterexample showing that the converse is not true.

4. Determine if the sequence of functions $f_n(x) = \sqrt{n}x^n(1-x)$ converges uniformly on $[0, 1]$.

5. Show that $\sum_{n=1}^{+\infty} \frac{\sin nx}{n}$ converges unifromly on $[\alpha, \beta] \subset (0, 2\pi)$.

6. Let $f(x, y) = \begin{cases} \frac{x^2y^3}{(x^2+y^2)^{2.1}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$
Is f differentiable at $(0, 0)$? Explain why.

7. Let f be differentiable at every point of R with values in R^2. Prove or disprove that if $a, b \in \mathrm{R}$, then there is a point c (lying between a, b) such that $f(b) - f(a) = Df(c)(b - a)$.

8. Let $uy + vx + w + x^2 = 0$ and $uvw + x + y + 1 = 0$. Show that we solve x, y in terms of u, v, w near $(u, v, w, x, y) = (2, 1, 0, -1, 0)$.

9. Evalute $\int\int_S (x^2 + y^2)dxdy$, where S is bounded by the hyperbolas $xy = 1$, $xy = 2$, $x^2 - y^2 = 1$, $x^2 - y^2 = 4$.

10. Let $S = \{(x, y, z) \in \mathrm{R}^3 | \mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2 = 1\}$. Evaluate the integral $\int\int_S (x^4 + y^4 + z^4)dA$, where dA is the surface area element in S.