

淡江大學 104 學年度碩士班招生考試試題

14

系別：數學學系 A 組

科目：基礎代數(含線性代數、代數學)

考試日期：3 月 8 日(星期日) 第 3 節

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1. (12%) Decide the dependence or independence of

(a) $(1,1,2), (1,2,1), (3,1,1)$;(b) $v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4 - v_1$ for any vectors v_1, v_2, v_3, v_4 ;(c) $(1,1,0), (1,0,0), (0,1,1), (x, y, z)$ for any numbers x, y, z .

2. (15%)

(a) $Ax = b$ has a solution under what conditions on b , if

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

(b) Find a basis for the nullspace of A .(c) Find the general solution to $Ax = b$, when a solution exists.(d) Find a basis for the column space of A .(e) What is the rank of A .

3. (12%) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}.$$

4. (11%) Let $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}$, find A^{-1} .5. (12%) If G is a group of even order, prove it has an element $a \neq e$ satisfying $a^2 = e$.

6. (12%) Show that every subgroup of an abelian group is normal.

7. (12%) Given an example of a non-abelian group in which $(xy)^3 = x^3y^3$ for all x and y .

8. (14%) Prove that any group of order 15 is cyclic.