

淡江大學 102 學年度碩士班招生考試試題

系別：數學學系

科目：基礎代數（含線性代數、代數學）

考試日期：3月10日(星期日) 第3節

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1. Decide whether each of the following sets of vectors is linearly dependent or linearly independent. (12%)

$$(a) \left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} \right\} \quad (b) \left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -1 \\ 4 \end{bmatrix} \right\} \quad (c) \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}$$

2. Let $A = \begin{bmatrix} 1 & -1 & 3 \\ 1 & 0 & -1 \\ 2 & 1 & 6 \end{bmatrix}$, find A^{-1} . (12%)

3. Let $A = \begin{bmatrix} 1 & -3 & 0 & 2 & 2 \\ -2 & 6 & 1 & 2 & -5 \\ 3 & -9 & -1 & 0 & 7 \\ -3 & 9 & 2 & 6 & -8 \end{bmatrix}$, find bases for $\text{null } A$, $\text{im } A$. (12%)

4. Compute (a) $\det(A)$ (b) $\det(\frac{1}{2}A)$ (c) $\det(\text{adj}A)$ (d) $\text{adj}(\text{adj}A)$, where

$$A = \begin{bmatrix} 4 & -1 & 3 & -1 \\ 3 & 1 & 0 & 2 \\ 0 & 1 & 2 & 2 \\ 1 & 2 & -1 & 1 \end{bmatrix}. \quad (14\%)$$

5. Let G be a group. Prove that G is abelian if $(ab)^2 = a^2b^2$ for every a, b in G . (12%)

6. Find the remainder when 10^{516} is divided by 7. (12%)

7. Show that Z_{18}^* is isomorphic to Z_{14}^* . (12%)

8. Show that each polynomial is irreducible in $\mathbb{Q}[x]$. (14%)

(a) $x^5 + 6x^4 + 9x^2 + 12x + 15$

(b) $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$