

淡江大學 100 學年度碩士班招生考試試題

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系別：數學學系

科目：線性代數 50% 及 代數學 50%

考試日期：2 月 28 日 (星期一) 第 3 節

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Linear Algebra

1. Find a basis of $U = \text{span}\{(1, -1, 3, 2), (0, -1, 2, 1), (2, 1, 0, 1)\}$. (10%)

2. Find bases for row space and column space of $\begin{bmatrix} 1 & 2 & 2 & -1 \\ 3 & 6 & 5 & 0 \\ 1 & 2 & 1 & 2 \end{bmatrix}$. (10%)

3. Let $T: R^4 \rightarrow R^3$, $T(x, y, z, w) = (x+y-z+2w, 2x+y+4z+3w, 5x+6y+z+9w)$. Find bases for kernel and range of T . (15%)

4. Decide whether each of the following sets of vectors is linearly dependent or linearly independent. (15%)

(a) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -1 \\ 4 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}$

(d) The columns of $\begin{bmatrix} 1 & 1 & 0 & -1 \\ 2 & 3 & 0 & 1 \\ -1 & 2 & 1 & 0 \end{bmatrix}$ as vectors in R^3 .

Algebra

5. Prove that every group of order 5 is abelian. (10%)

6. Suppose that a group G has subgroups of order 25 and 55.

If $200 \leq |G| \leq 300$, determine $|G|$. (10%)

7. Let $\alpha: G_1 \rightarrow G_2$ be a group homomorphism, prove that (20%)

(a) $\alpha(G_1) \leq G_2$

(b) $\ker \alpha \triangleleft G_1$

(c) $\ker \alpha$ is trivial $\Leftrightarrow \alpha$ is 1-1.

8 Show that a group G is abelian if and only if $(xy)^2 = x^2y^2$ for all x, y in G . (10%).