淡江大學99學年度碩士班招生考試試題

系別：數學學系　　　　　　科目：線性代數 50% 及 代 數 學 50%

SHOW YOUR WORK

1. Solve $x + 2y - z = 2$
   $2x + 5y - 3z = 1$
   $x + 4y - 3z = 3$ **by Gaussian elimination.** (10%)

2. Let A = $\begin{bmatrix} 2 & 7 & 1 \\ 1 & 4 & -1 \\ 1 & 1 & 0 \end{bmatrix}$,   Find A^{-1}. (10%)

3. Let T: $R^4 \to R^3$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 8x_2 - 7x_3 - 4x_4 \\ 2x_1 - 3x_2 - x_3 + 5x_4 \\ 3x_1 + 2x_2 + 5x_3 + 14x_4 \end{bmatrix}$. Find bases for

   kerT and ImT. (15%)

4. (a) Find an orthonormal basis for the subspace W = {(x, y, z) | x + 3y - 2z = 0} of R^3.

   (b) Find the orthogonal projection of $\vec{X} = (2, 1, 3)$ on the subspace W. (15%)

5. Show that $f(x) = x^5 + 6x^4 + 12x + 15$ is irreducible in Q[x]. (10%)

6. If G is a group in which $(ab)^3 = a^3b^3$, $(ab)^4 = a^4b^4$, $(ab)^5 = a^5b^5$ for all $a, b \in G$, show that G is abelian. (10%)

7. Let G be a nonempty set closed under an associative product, which in addition satisfies:

   (a) There exists an $e \in G$ such that $ea = a$ for all $a \in G$.

   (b) Given $a \in G$, there exists an element $b \in G$ such that $ba = e$.

   Show that G must be a group under this product. (10%)

8. Construct a field of order 4. (10%)

9. Let F = Q, E = Q($\sqrt{2}, \omega$), where $\omega = e^{\frac{2}{3}\pi i} = \dfrac{-1 + \sqrt{-3}}{2}$. Find the Galois group of E over F. (10%)