Partial credit—You must show all your work.

1. (10 %) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 4 \\ 1 & 1 & 5 \end{bmatrix}$. Find a basis for the row space of A.

2. (15 %) Let $A = \begin{bmatrix} -1 & -2 & 11 \\ 1 & 3 & -15 \\ 0 & -1 & 5 \end{bmatrix}$. Find A^{-1}.

3. (15 %) Use the Gram-Schmidt process to generate an orthogonal set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \}$ from the given linearly independent vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.
(a) (15 %) Find an invertible matrix S such that $S^{-1}AS$ is diagonal.
(b) (5 %) Calculate A^5.

5. $T : P_3 \to P_4$ is defined by $T(p) = p'(0)$, where P_3 is the set of all polynomials of degree at most 3 with basis $B = \{1, x, x^2, x^3\}$ and P_2 is the set of all polynomials of degree at most 2 with basis $C = \{1, x, x^2, x^3, x^4\}$.
(a) (10 %) Show that T is a linear transformation.
(b) (10 %) Find the matrix for T with respect to B and C.

6. (20 %) Let $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & 0 & 2 & -2 \\ 3 & -1 & 1 & 1 \\ 2 & 0 & -1 & 2 \end{bmatrix}$. Find $\det(A)$.