23-1

淡江大學96學年度碩士班招生考試試題

系別：數學學系

科目：線 性 代 數

准帶項目請打「V」	
	簡單型計算機

本試題共 2 頁

Show your work

1. Find A^{-1} if $A=\begin{bmatrix}2 & 7 & 1\\ 1 & 4 & -1\\ 1 & 3 & 0\end{bmatrix}$. (10%)

2. Compute the rank of $A=\begin{bmatrix}1 & 2 & 2 & -1\\ 3 & 6 & 5 & 0\\ 1 & 2 & 1 & 2\end{bmatrix}$ and find bases for the row space and the column space of A. (10%)

3. Diagonalize the matrix $\begin{bmatrix}2 & 0 & 0\\ 1 & 2 & -1\\ 1 & 3 & -2\end{bmatrix}$. (10%)

4. Find an orthogonal basis of the row space of $\begin{bmatrix}1 & 1 & -1 & -1\\ 3 & 2 & 0 & 1\\ 1 & 0 & 1 & 0\end{bmatrix}$. (10%)

5. Let $T: V\to W$ be a linear transformation and let $\{\vec{e_1},\vec{e_2},\cdots\vec{e_r},\vec{e_{r+1}},\cdots,\vec{e_n}\}$ be a basis of V such that $\{\vec{e_{r+1}},\cdots,\vec{e_n}\}$ is a basis of the kernel of T, prove that $\{T(\vec{e_1}),\ \cdots,\ T(\vec{e_r})\}$ is a basis of the image of T.. (10%)

6. Find a basis of U = span{(1, -1, 3, 2), (0, -1, 2, 1), (2, 1, 0, 1),(3, 5, -3,0)}. (10%)

7. Decide whether each of the following sets of vectors is linearly dependent or linearly independent. (20%)

A. $\left\{\begin{bmatrix}1\\-1\\3\end{bmatrix},\begin{bmatrix}1\\1\\-2\end{bmatrix},\begin{bmatrix}3\\-1\\4\end{bmatrix}\right\}$

B. $\left\{\begin{bmatrix}-1\\-1\\2\\-2\end{bmatrix},\begin{bmatrix}1\\2\\1\\1\end{bmatrix},\begin{bmatrix}3\\3\\-1\\4\end{bmatrix}\right\}$

C. $\left\{\begin{bmatrix}1\\2\\3\end{bmatrix},\begin{bmatrix}0\\0\\0\end{bmatrix},\begin{bmatrix}4\\5\\6\end{bmatrix}\right\}$

本試題雙面印製

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D. The columns of $\begin{bmatrix} 1 & 1 & 0 & -1 \\ 2 & 3 & 0 & 1 \\ -1 & 2 & 1 & 0 \end{bmatrix}$ as vectors in R^3.

8. Let $\{\vec{u}, \vec{v}, \vec{w}\}$ be linearly independent, prove that $\{\vec{u}+\vec{v},\ \vec{v}+\vec{w},\ \vec{w}+\vec{v}\}$ is also linearly independent. (10%)

9. Let $\{\vec{v_1}, \vec{v_2}, \cdots, \vec{v_n}\}$ be a set of vectors in a vector space U. Prove that $\{\vec{v_1}, \vec{v_2}, \cdots, \vec{v_n}\}$ is linearly dependent if and only if one of $\vec{v_1}, \vec{v_2}, \cdots, \vec{v_n}$ is a linear combination of the others. (10%)