

系別：數學學系

科目：線性代數

准帶項目請打「V」	
	簡單型計算機

本試題共 1 頁

95 年度碩士班線代試題

1. If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation and $T(1, 3) = (1, 1)$, $T(1, 1) = (0, 1)$, find $T(x, y)$. (10%)

2. Prove the Dimension Theorem:

Let $T: V \rightarrow W$ be any linear transformation and assume that $\ker T$ and $\text{im} T$ are both finite dimensional. Then V is also finite dimensional and

$$\dim V = \dim(\ker T) + \dim(\text{im} T). \quad (15\%)$$

3. Which of the following sets of vectors in \mathbb{R}^3 is linearly independent? (15%)

- (A) $\{(2, -6, 8), (-3, 9, -12)\}$
- (B) $\{(1, 3, 5), (0, 0, 0), (2, 4, 6)\}$
- (C) $\{(2, 5, 7), (1, -1, 4), (-3, 6, 19), (45, 62, -78)\}$
- (D) $\{(1, 2, 3), (2, 3, 8), (5, 8, 19)\}$
- (E) $\{(1, 1, 1), (1, 2, 3), (2, -1, 1)\}$

4. Let $A = \begin{bmatrix} 1 & -2 & 5 & -3 \\ 2 & 3 & 1 & -4 \\ 3 & 8 & -3 & -5 \end{bmatrix}$, which of the following is true? (15%)

(A) $\{(1, -2, 5, -3), (0, 7, -9, 2)\}$ is a basis for the row space of A

(B) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 8 \end{bmatrix} \right\}$ is a basis for the column space of A

(C) The row space of A is a 3-dimensional subspace of \mathbb{R}^4

(D) The column space of A is a 2-dimensional subspace of \mathbb{R}^3

(E) The rank of A is 2

5. Let A be a real square matrix (A is $n \times n$) (10%)

- (1) What is the sufficient and necessary condition for A to be diagonalizable?
- (2) What is the sufficient and necessary condition for A to be orthogonally diagonalizable?

6. Let A be a complex square matrix (A is $n \times n$). What is the sufficient and necessary condition for A to be unitarily diagonalizable? (10%)

7. Identify the curve of the quadratic equation $2x^2 - 4xy + 5y^2 - 36 = 0$. (5%)

8. Let A and B denote invertible $n \times n$ matrices. Show that: (20%)

- (1) $\text{adj}(\text{adj} A) = (\det A)^{n-2} A$ ($n \geq 2$)
- (2) $\text{adj}(A^{-1}) = (\text{adj} A)^{-1}$
- (3) $\text{adj}(AB) = (\text{adj} B)(\text{adj} A)$