

淡江大學九十四學年度碩士班招生考試試題 ²⁸¹

系別：數學學系

科目：線性代數

准帶項目請打「V」

X 簡單型計算機

本試題共 / 頁

Show Your Work

- Determine which of the following sets are linearly independent:
 - $\{(1,3,2), (2,6,4)\}$
 - $\{(4,2,1), (2,6,4), (0,0,0)\}$
 - $\{(1,2,1), (2,3,1), (-1,2,-3)\}$
 - $\{(2,5,1), (1,1,-1), (0,2,3), (2,2,3)\}$
 - $\{(7,1,2,0), (8,0,1,-1), (1,0,0,-2)\}$ (15%)
- Let $S = \{(1,3,2,-1), (-2,-6,-4,2), (2,4,-1,0), (0,1,0,-1), (1,3,1,-1)\}$. If $W = \text{span}(S)$, find a subset B of S that is a basis for W . (10%)
- Prove that there exists a basis for any vector space. (10%)
- Let $T: V \rightarrow W$ be a linear transformation of vector spaces. If $\dim V = n < \infty$, prove that $n = \dim(\ker T) + \dim(\text{im } T)$, where $\text{im } T$ is the image of T . (10%)
- Find the minimal solution to the system

$$\begin{aligned} x + 2y + z &= 4 \\ x - y + 2z &= -11 \\ x + 5y &= 19 \end{aligned}$$
 (10%)
- Let T be the linear operator on C^2 defined by $T(a, b) = (2ia + 3b, a - 4ib)$. Find $T^*(a, b)$, where $i = \sqrt{-1}$. (5%)
- Let V and W be finite-dimensional vector spaces over a field F of dimension n and m , respectively. Let $L(V, W)$ denote the set of all linear transformations from V into W . Prove that $L(V, W)$ is a vector space over F of dimension mn . (10%)
- Find the Jordan canonical form for $A = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{bmatrix}$. (10%)
- For $A = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$, find a matrix P such that $P^{-1}AP$ is diagonal. (10%)
- Let V be an inner product space, and T be a normal operator on V . Prove that

$$\|T(\vec{x})\| = \|T^*(\vec{x})\|. \quad (10\%)$$