淡江大學九十三學年度碩士班招生考試試題

系別：數學學系　　科目：線　性　代　數

准帶項目請打「○」否則打「×」
× 簡單型計算機

Answer all questions. Show all work.

1. Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be given by
$$T(x, y, z) = (x + y + z, x + y + z, x + y + z).$$
Find the characteristic and minimal polynomials of T. (10%)

2. Let $A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 5 & -3 \\ -1 & 2 & 4 \end{pmatrix}$. Find (i) $adj(A)$; (ii) A^{-1}. (12%)

3. Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Find (i) the eigenvalues and eigenvectors of A; (ii) A^n where n is a positive integer. (14%)

4. Find a basis for the orthogonal complement in $\mathbf{R}^4$ of the subspace $W = sp((1, 2, 2, 1), (3, 4, 2, 3))$. (10%)

5. Let $B = (x^2, x, 1)$ and $B' = (x^2 - x, 2x^2 - 2x + 1, x^2 - 2x)$ be ordered bases of $P_2 = \{\textit{polynomials over } \mathbf{R} \textit{ with degree at most } 2\} = \{a_2x^2 + a_1x + a_0 \mid a_2, a_1, a_0 \in \mathbf{R}\}$. Find the coordinate matrix from B to B', and use it to find the coordinate vector of $2x^2 + 3x - 1$ relative to B'. (12%)

6. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$. Factor A in the form $A = QR$,where Q is a 3×2 matrix with orthonormal column vectors and R is a 2×2 upper triangular invertible matrix. (12%)

7. Suppose $T : V \to V$ is a linear transformation in the finite dimensional vector space V. Show that there exists an integer k such that $Im(T^j) = Im(T^k)$, $ker(T^j) = ker(T^k)$ whenever $j \geq k$, and show that $ker(T^k)$ and $Im(T^k)$ are a complementary pair of T-invariant subspaces of V. (10%)

8. Let $T : V \to V$ be a self adjoint linear transformation in the finite dimensional inner product space V. Suppose that e_1 and e_2 are distinct eigenvalues of T with corresponding eigenvectors $\mathbf{E}_1$ and $\mathbf{E}_2$. Show that $\mathbf{E}_1$ and $\mathbf{E}_2$ are orthogonal. (10%)

9. Let $f(\mathbf{x})$ be a quadratic form, and let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of the symmetric coefficient matrix of $f(\mathbf{x})$. Show that the maxima of $f(\mathbf{x})$ on $\| \mathbf{x} \| = 1$ is the maximum of the λ_i. (10%)