淡江大學九十二學年度碩士班招生考試試題

系別：數學學系　　　　科目：線 性 代 數

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本試題共 二 頁

本試題雙面印製

SHOW ALL YOUR WORKING

1. Let A be the 3×3 matrix given by A = PQ, where

$$P = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \text{ and } Q = [2\ -1\ 2].$$

Find A, A^2 and A^{100}. (13%)

2. Given $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & -1 & 3 \end{bmatrix}$, evaluate det A and find A^{-1} if it exists. (12%)

3. Let $\mathbb{P}$ be the set of all polynomials in one variable with real coefficients. Then $\mathbb{P}$ is a real vector space under usual polynomial addition and scalar multiplication. Let $\mathbb{P}_n$ be the subspace consisting of polynomials of degree less than or equal to n. Let T: $\mathbb{P}_2(\mathbb{R}) \longrightarrow \mathbb{P}_3(\mathbb{R})$ be the linear transformation defined by: $T(p(x)) = 2p(x) + p'(x)$, where $p'(x)$ denotes the derivative of $p(x)$.
(i) Show that $\{1, 1+x, (1+x)^2\}$ is a basis of $\mathbb{P}_2(\mathbb{R})$. (5%)
(ii) Find the matrix representation of T relative to the ordered bases $\{1, 1+x, (1+x)^2\}$ of $\mathbb{P}_2(\mathbb{R})$ and $\{1, x, x^2, x^3\}$ of $\mathbb{P}_3(\mathbb{R})$. (5%)
(iii) What is the rank of T ? (4%)
(iv) Is T one-to-one ? Onto ? (6%)

4. Let W_1 and W_2 be subspaces of a vector space V. Prove that if $W_1 \cup W_2$ is also a subspace of V, then either $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. (15%)

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5. Given matrices

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & x \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

Suppose that A and B are similar.

(a) Determine x and y. (10%)

(b) Find an invertible matrix P such that $P^{-1}AP = B$. (10%)

6. (a) Find the solution set of the following system of linear equations:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= 0 \\ x_1 + x_2 + x_3 + 2x_4 + 2x_5 &= 3 \\ x_1 + x_2 + x_3 + 2x_4 + 3x_5 &= 2 \end{aligned} \quad (10\%)$$

(b) Let W be the subspace of R^5 spanned by $(1,1,1,1,1)^T$, $(1,1,1,2,2)^T$ and $(1,1,1,2,3)^T$. Find an orthonormal basis for the subspace $W^\perp$. (10%)

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