

## 淡江大學九十一年度碩士班招生考試試題

系別：數學系

科目：線性代數

准帶項目請打「○」否則打「×」	
計算機	字典
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本試題共 1 頁

Show all your works

1. Let  $A = \begin{bmatrix} -1 & -1 & 1 & 0 \\ 2 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 1 & 3 & -1 & 2 \end{bmatrix}$ .

- (a) Evaluate the determinant of  $A$  by expanding along the second column. (5%)  
 (b) Find the inverse matrix of  $A$  if it exists. (10%)

2. Find a basis of the solution space of  $AX = 0$ , where  $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -3 & 2 \end{bmatrix}$ . (15%)

3. Let  $\mathbb{P}$  be the set of all polynomials in one variable with real coefficients. Then  $\mathbb{P}$  is a real vector space under usual polynomial addition and scalar multiplication. Let  $\mathbb{P}_n$  be the subspace consisting of polynomials of degree less than or equal to  $n$ .

- (a) If  $a$  is a real number, let  $W = \{p(x) \in \mathbb{P}_n : p(a) = 0\}$ , show that  $W$  is a subspace of  $\mathbb{P}_n$ . (5%)  
 (b) Show that  $\{(x-a), (x-a)^2, \dots, (x-a)^n\}$  is a basis of  $W$ . (10%)  
 (c) Let  $T : \mathbb{P}_3 \rightarrow \mathbb{P}_3$  be the map defined by  $T(p(x)) = 2p(x) - 3p'(x)$ . Show that  $T$  is a linear transformation. (5%)  
 (d) Find the matrix representation  $A$  of  $T$  with respect to the basis  $\{1, x, x^2, x^3\}$ . (5%)  
 (e) Let  $B$  be the matrix representation of  $T$  with respect to the basis  $\{1+x, x-x^2, x^2+x^3, -2+x^3\}$ . Find an invertible matrix  $U$  satisfying  $B = U^{-1}AU$ . (5%)

4. Let  $A = \begin{bmatrix} -1 & -1 & 1 & 0 \\ 2 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 1 & 3 & -1 & 2 \end{bmatrix}$ .

- (a) Find the characteristic polynomial of  $A$ . (5%)  
 (b) Find the eigenvalue of  $A$  and the corresponding eigenspaces. (10%)  
 (c) Find the Jordan canonical form of  $A$ . (5%)  
 (d) Compute  $A^{100}$ . (5%)

5. Show that any two bases of a finite dimensional vector space have the same cardinality. (15%)