

淡江大學九十學年度碩士班招生考試試題

系別：數學學系

科目：線性代數

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本試題共 / 頁

SHOW ALL YOUR WORK

1. Let $A = \begin{pmatrix} 5 & 4 & 3 \\ -1 & 0 & -3 \\ 1 & -2 & 1 \end{pmatrix}$, find the eigenvalues and eigenvectors of A. Is A

diagonalizable? Give your reason. (15%)

2. Find the matrix representation of the following linear transformation

$$T: R^4 \rightarrow P_3(R) \text{ given by}$$

$$T(a_1, a_2, a_3, a_4) = a_2 + a_3 + (a_1 + a_4)x + (a_2 + a_3)x^2 + (a_1 - a_4)x^3$$

relative to the ordered bases

$$A_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, A_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, A_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, A_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ for } R^4$$

and $B_1 = 1 + x, B_2 = 1 - x, B_3 = 1 - x^2, B_4 = 1 + x^3$ for $P_3(R)$ is

the set of all polynomials of degree less or equal to 3 with real coefficients.

What is the rank of T? Give your reason. (15%)

3. Let A be a real n by n upper or lower triangular matrix satisfying

$$AA' = A'A, \text{ where } A' \text{ is the transpose of } A. \text{ Show that } A \text{ is a diagonal matrix.}$$

(10%)

4. Let $w \in R^n$ with $ww' = 1$. If $A = I - 2ww'$, show that A is symmetric and orthogonal. (10%)

5. Let V be a finite dimensional inner product space and W be a linear subspace of V. Show that $V = W + W^\perp$. (15%)

6. Let $T: V \rightarrow W$ be any linear transformation and assume that $\text{Ker}(T)$ and $\text{Im}(T)$ are both finite dimensional (20%)

a) Show that V is also finite dimensional and $\dim(V) = \dim(\text{Ker}(T)) + \dim(\text{Im}(T))$

b) If V is the space of all n by n real matrices, $W = R$ and $T(A) = \text{trace}(A)$. Show that $\dim(\text{Ker}(T)) = n^2 - 1$

7. Let A be a real n by n matrix and b be a real n by 1 vector. Show that $Ax = b$ has a solution if and only if $ab = 0$ for all 1 by n vectors a such that $aA = 0$. (15%)