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淡江大學八十九學年度碩士班招生考試試題

系別：數學學系　　　　　科目：線性代數

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Answer all questions. Show all work.

1. Find A^{-1} if $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 2 & 3 & -1 \end{pmatrix}$.　　(10%)

2. Let $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$.

   (a) Find the eigenvalues of A.

   (b) Find a matrix P such that $P^{-1}AP$ is a diagonal matrix.

   (18%)

3. Let W and U be subspaces of a vector space V, and let $W \cap U = \{\mathbf{o}\}$. Let $\{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k\}$ be a basis for W, and let $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m\}$ be a basis for U. Prove that, if each vector $\mathbf{v}$ in V is expressible in the form $\mathbf{v} = \mathbf{w} + \mathbf{u}$ for $\mathbf{w} \in W$ and $\mathbf{u} \in U$, then

$$\{\mathbf{w}_1, \ldots, \mathbf{w}_k, \mathbf{u}_1, \ldots, \mathbf{u}_m\}$$

   is a basis for V.　　(10%)

4. If A is a nonsingular symmetric matrix, show that A^{-1} is symmetric. (10%)

5. Let V and V' be vector spaces and let $T : V \to V'$ be a linear transformation. Prove that T is one-to-one $\Leftrightarrow$ $Ker(T) = 0$.　　(12%)

6. Let V and W be finite dimensional inner product spaces with $dim\ V \leq dim\ W$. Show that there exists an isometric embedding $T : V \to W$. (10%)

7. Suppose $T : V \to V$ is a projection, i.e., $T^2 = T$. Show that the only eigenvalues of T are 0 and 1.　　(10%)

8. Let A be an $n \times n$ matrix such that $A\mathbf{x} \cdot A\mathbf{y} = \mathbf{x} \cdot \mathbf{y}$ for all $\mathbf{x}$ and $\mathbf{y}$ in $\mathbf{R}^n$. Show that A is an orthogonal matrix.　　(10%)

9. Find the rank of

$$A = \begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 2 & 1 & -1 & 2 & 0 \\ -1 & 2 & 1 & 1 & 3 \\ 3 & -7 & 8 & 9 & 13 \\ 1 & 5 & -8 & -5 & -12 \end{pmatrix}.$$

   (10%)