

# 淡江大學八十八學年度碩士班招生考試試題

系別：數學學系

科目：線性代數

本試題共 1 頁

1. a) Let  $V$  be a finite dimensional vector space and  $T: V \rightarrow V$  be a linear transformation satisfying  $T^2 = T$ . Show that  $V = \text{Ker}T \oplus \text{Im}T$ . (10 points)

b) Let  $P_2$  be the space of polynomials of degree less than 2 over  $\mathbb{R}$ . Let  $T: P_2 \rightarrow P_2$  be defined by

$$T(a + bx + cx^2) = (a + b - c)(1 + x + x^2)$$

Show that  $T^2 = T$  and find the matrix representation of  $T$ . (10 points)

2. Let  $N$  be an  $n$  by  $n$  strictly lower triangular matrix.

a) Show that  $N^n = O$ , where  $O$  is the  $n$  by  $n$  zero matrix. (5 points)

b) Show that the matrix  $I + N$  is invertible, where  $I$  is the  $n$  by  $n$  identity matrix. (10 points)

3. Let  $V$  and  $W$  be finite dimensional vector space. Show that

$\dim V = \dim W$  if and only if  $V$  and  $W$  are isomorphic. (20 points)

4. Let  $V$  be the space of all functions  $f$  from  $\{1, 2, \dots, n\}$  to  $\mathbb{R}$ . If  $T: V \rightarrow \mathbb{R}^n$  is defined by  $T(f) = (f(1), f(2), \dots, f(n))$ , show that  $T$  is an isomorphism.

(10 points)

5. Let  $A$  be an  $n$  by  $n$  real symmetric matrix

a) Show that all the eigenvalues of  $A$  are real. (5 points)

b) Show that there exists  $n$  eigenvectors of  $A$  that form an orthonormal set. (10 points)

c) Suppose the  $A$  is a positive definite matrix, show that there exists a positive definite matrix  $B$  such that  $A = B^2$ . (10 points)

6. Consider the quadratic equation

$$5x^2 - 8xy + 5y^2 = 9$$

Find a rotation so that the equation has no cross term and sketch the graph.

(10 points)