

淡江大學 98 學年度碩士班招生考試試題

1

系別：數學學系

科目：代 數 學

准帶項目請打「V」

簡單型計算機

本試題共 / 頁， / 大題

Show your work.

1. Show that a group G is abelian if and only if $(x y)^2 = x^2 y^2$ for all x, y in G . (10%)
2. Prove Z_{18} is cyclic and find all distinct generators. (10%)
3. Let $Z_{18}^* = \{1, 5, 7, 11, 13, 17\}$ be the multiplicative group of integers modulo 18, prove that Z_{18}^* is cyclic. (10%)
4. Up to isomorphism, find all abelian groups of order 36. (10%)
5. Up to isomorphism, find all groups of order 15. (10%)
6. Up to isomorphism, find all groups of order 14. (10%)
7. If p is a prime, prove that $a^p - a$ is divisible by p for all integers a . (10%)
8. Let R be a ring with unity 1. If $r^3 = r$ for all r in R , Show that R is commutative. (10%)
9. Let R be a commutative ring with unity 1. Prove that an ideal M is maximal if and only if R/M is a field. (10%)
10. Let $f: G \rightarrow G'$ be a group homomorphism, prove that $\ker f$ is a normal subgroup of G . (10%)