淡江大學97學年度碩士班招生考試試題

系別：數學學系　　科目：代　數　學

本試題共 1 頁，9 大題

1. Find the remainder when:
   (a)10^{516} is divided by 7
   (b)481^{5251} is divided by 53　(10%)
2. Let G be the rational numbers except -1.
   Show that $(G,*)$ is a group where $a*b=a+b+ab$ for all $a,b\in G$.
   (10%)
3. In each case determine all subgroups of G and draw the lattice diagram.
   (a)$G = Z_{45}$ (b) $G = Z_{18}^{*}$, where $Z_{18}^{*} = \{a\in Z_{18} \mid a \text{ and } 18 \text{ are relatively prime}\}$ (c) $G = S_3$　(20%)
4. Prove that every finite integral domain is a field.　(10%)
5. Let G be a group and
   $$(ab)^3 = a^3b^3$$
   $$(ab)^4 = a^4b^4$$
   $$(ab)^5 = a^5b^5$$
   for all a and b in G.
   Show that G is abelian.　(10%)
6. Show that a group G is abelian if $x^2 = e$ for all x in G. Give an example showing that the converse is false.　(10%)
7. Find all groups of order 77 up to isomorphism.　(10%)
8. Find all groups of order 55 up to isomorphism.　(10%)
9. Find the Galois group of $x^2 - 2$ over Q.　(10%)