淡江大學 95 學年度碩士班招生考試試題

系別：數學學系　　　　　　科目：代　數　學

准帶項目請打「V」	
	簡單型計算機

本試題共 1 頁

Answer all questions. Show all work.

1. Let $\phi : G \to G'$ be a group homomorphism. Let $|G|$ and $|\phi(G)|$ be the order of $|G|$ and order of $|\phi(G)|$ respectively. If $|G|$ is finite, show that $|\phi(G)|$ is finite and is a divisor of $|G|$. (10%)

2. Let $\phi : G \to H$ be a group homomorphism and let $K = \{a \in G | \phi(a) = e_H\}$ where e_H is the identity element of H. Prove that K is a normal subgroup of G. (10%)

3. Let $\phi : \mathbf{Z}_{18} \to \mathbf{Z}_{12}$ be the homomorphism where $\phi(1) = 10$
   (a) Find the kernel of ϕ.
   (b) Lists the cosets in $\mathbf{Z}_{18}/ker(\phi)$, showing the elements in each coset.
   (c) Find the group $\phi(\mathbf{Z}_{18})$.

   (15%)

4. Let $\mathbf{R}$ be the field of real numbers.
   (a) Prove that the set $T = \{\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} | a, b \in \mathbf{R}\}$ is a subring of $M(R)$ where $M(R)$ is the ring of 2×2 matrices with entries in $\mathbf{R}$.
   (b) Prove that the set $I = \{\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} | b \in \mathbf{R}\}$ is an ideal in the ring T.
   (c) Show that every coset in T/I can be written in the form $\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} + T$.

   (18%)

5. Let $\mathbf{Q}$ be the field of rational numbers.
   (a) Prove that $\mathbf{Q}(\sqrt{2}) = \{r + s\sqrt{2} | r, s \in \mathbf{Q}\}$ is a subfield of $\mathbf{R}$.
   (b) Show that $\mathbf{Q}(\sqrt{2})$ is isomorphic to $\mathbf{Q}[x]/(x^2 - 2)$.

   (15%)

6. Let $\mathbf{C}$ be the field of complex numbers. Prove that the function $f : \mathbf{C} \to \mathbf{C}$ which is defined by $f(a + bi) = a - bi$ for any $a + bi \in \mathbf{C}$ is an isomorphism. (12%)

7. Let be the ring of integers. If p is prime integer and $M = \{(pa, b) | a, b \in \mathbf{Z}\}$, prove that M is a maximl ideal in $\mathbf{Z} \times \mathbf{Z}$. (10%)

8. Show that $x^2 - 3$ and $x^2 - 2x - 2$ are irreducible in $\mathbf{Q}[x]$ and find their splitting fields. (10%)