

29

211

淡江大學九十四學年度碩士班招生考試試題

系別：數學學系

科目：代 數 學

准帶項目請打「V」

簡單型計算機

本試題共 / 頁

1. (10 points) Let  $\mathbb{F}$  be a field. Show that every ideal in  $\mathbb{F}[x]$  is principle.
2. (10 points) Find an irreducible polynomial of degree 3 over  $\mathbb{Z}_3$  and construct a field of 27 elements.
3. (10 points) Show that a group of order 2005 is cyclic.
4. (10 points) Let  $\varphi : G \rightarrow G'$  be a group homomorphism. Suppose that  $|G| = 24$ ,  $|G'| = 30$  and  $\varphi$  is not the trivial homomorphism. Find possible orders of image of  $\varphi$ .
5. (10 points) List all abelian groups of order 360.
6. (10 points) Let  $\varphi : \mathbb{F} \rightarrow \mathbb{F}'$  be a homomorphism between two fields. Show that if  $\varphi$  is not the zero homomorphism, then  $\varphi$  is one to one.
7. (15 points)  
Let  $R = \mathbb{Z}[\sqrt{-1}] = \{a + b\sqrt{-1} \mid a, b \in \mathbb{Z}\}$ .
  - (a) Determine whether  $3 + 2\sqrt{-1}$ ,  $4 + 3\sqrt{-1}$  are primes.
  - (b) Let  $p$  be a prime integer in  $\mathbb{Z}$ . Show that  $p$  is a prime in  $R$  if and only if  $p$  can not be written as  $a^2 + b^2$  for any integer  $a, b \in \mathbb{Z}$ .
  - (c) Let  $I \subset R$  be an ideal. Show that  $R/I$  is finite.
8. (25 points)  
Let  $\zeta$  be a primitive 7-th root of unity.
  - (a) Find order of  $\text{Gal}(\mathbb{Q}(\zeta)/\mathbb{Q})$  and write explicitly its elements.
  - (b) Find an intermediate field  $K$  of  $\mathbb{Q}(\zeta)$  such that  $[\mathbb{Q}(\zeta) : K] = 2$ .
  - (c) Find the splitting field of  $x^7 - 2$  and find its degree of extension over  $\mathbb{Q}$ .