淡江大學九十三學年度碩士班招生考試試題

系別：數學學系　　科目：代　數　學

准帶項目請打「○」否則打「×」
簡單型計算機
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Show your work. 10 points each.

1. Show that $Q(\sqrt{2}) = \{ a + b\sqrt{2}; a, b \in Q\}$ is a field, where Q is the set of rational numbers.
2. Describe the Galois group $Gal(Q(\sqrt{2})/Q)$.
3. Up to isomorphism, find the groups of order 8.
4. Up to isomorphism, find the abelian groups of order 72.
5. Prove that A_4 is not simple. Is A_5 simple?
6. If N and K are normal subgroups of a group G, show that $HK = \{ hk ; h \in H, k \in K\}$ is also a normal subgroup of G.
7. Show that every group of order 6 is either cyclic or isomorphic to S_3.
8. State the Fundamental Theorem of Galois Theory.
9. Are the additive group Z and Q isomorphic? Support your answer.
10. Let R be a commutative ring with identity. Show that R is a prime ideal if and only if R/P is an integral domain.