淡江大學九十一學年度碩士班招生考試試題

系別：數學系

科目：代 數 學

准帶項目請打「○」否則打「×」	
計算機	字典
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1. (20 points)

   (a) Shou that $\mathbb{Z}$ is a principle ideal domain.

   (b) Find the ideal $(1365) + (1430)$.

   (c) Find all ideals in $\mathbb{Z}_{72}$.

   (d) Find all units in $\mathbb{Z}_{72}$.

2. (20 points)

   Let G be a group and H, K be subgroups og G. Suppose that H is normal in G.

   (a) Show that HK is a subgroup of G.

   (b) Show that HK/H is isomorphic to $K/H \cap K$.

3. (10 points)

   Let ζ be a primitive 5-th root of unity. Show that $\mathrm{Gal}_{\mathbb{Q}}\mathbb{Q}(\zeta)$ is a cyclic group of order 4.

4. (10 points)

   Let L be a finite extension of K and D be a subring such that $K \subset D \subset L$. Show that D is a field.

5. (20 points)

   Let $R = \mathbb{Z}[\sqrt{-1}] = \{a + b\sqrt{-1} \mid a, b \in \mathbb{Z}\}$.

   (a) Determine whether $1 + 3\sqrt{-1}$, $2 + 3\sqrt{-1}$ are primes.

   (b) Let p be a prime integer in $\mathbb{Z}$. Show that p is a prime in R if and only if p can not be written as $a^2 + b^2$ for any integer $a, b \in \mathbb{Z}$.

   (c) Let $I \subset R$ be an ideal. Show that R/I is finite.

6. (10 points)

   Show that a group of order 6 is either cyclic or isomorphic to S_6.

7. (10 points)

   Let R be a commutative ring with identity. Show that R is a field if and only if (0) is a maximal ideal in R.