淡江大學八十九學年度碩士班招生考試試題

系別：數學學系　　　　科目：代　數　學

本試題共　/　頁

每題10分.

1. State the definitions of the following: ① group ② cyclic group ③ ring.

2. Up to isomorphism, find all abelian groups of order 500.

3. Let G be a group in which $(ab)^3 = a^3b^3$ and $(ab)^5 = a^5b^5$ for all $a, b \in G$. Show that G is abelian.

4. Show that a group of order 36 has a normal subgroup of order 3 or 9.

5. Let G be a group and H a normal subgroup of G. If G/H is abelian, prove that $aba^{-1}b^{-1} \in H$ for all $a, b \in G$.

6. Let R be a ring. If $r^3 = r$ for all $r \in R$, show R is commutative.

7. Show that $\mathbb{Z}(\sqrt{5}) = \{m + n\sqrt{5} \mid m, n \in \mathbb{Z}\}$ is not a UFD by showing that $1+\sqrt{5}$ is an irreducible that is not prime.

8. Let $E = \mathbb{Q}(\sqrt[4]{2}, i)$, where $\mathbb{Q}$ is the field of rational numbers. Find the Galois group of E over $\mathbb{Q}$.

9. Let $F \subseteq E$ be a radical extension, where char $F = 0$. Prove that $Gal(E:F)$, the Galois group of E over F, is solvable.

10. Prove that any finitely generated abelian group is the direct sum of a finite number of cyclic groups.