淡江大學八十七學年度碩士班入學考試試題

系別： 數學系　　科目： 代數學

1. Let $f : G \longrightarrow G'$ be a homomorphism of finite groups. Suppose that the orders of G and G' are relatively prime. Prove that the image of f is trivial. (10%)

2. Let Z be the center of a group G. Suppose that G/Z is cyclic. Show that G is abelian. (10%)

3. Let G be a group of order 187.
(a) Show that there is one and only one subgroup of G whose order is 17. (5%)
(b) Show that there is one and only one subgroup of G whose order is 11. (5%)
(c) Show that G is abelian. (5%)

4. Let G be a cyclic group of order m.
(a) Find all the homomorphisms of G into itself. (5%)
(b) Find all the bijective homomorphisms of G into itself. (5%)

5. Show that every ideal of an Euclidean domail is principal. (10%)

6. Let p be a prime number. Let $R = \{\frac{b}{a} \in Q : p \text{ does not divide } a.\}$. Let U be the group of units in R.
(a) Show that the subset $R - U$ is an ideal in R. (5%)
(b) Show that R has a unique maximal ideal. (5%)

7. Determine which of the following polynomials are irreducible over the rational numbers. (a) $x^3 - x + 1$ (b) $x^5 + 2$ (10%)

8. Let n be a positive integer. Let K be the splitting field of $x^n - 1$ over the rational numbers Q. Let ξ be a primtive nth-root of unity.
(a) Find all primitive nth-root of unity. (5%)
(b) Find the Galois group of K over Q. (5%)

9. Let K be the splitting field of $x^3 - 6$ over the rational numbers Q. Let G be the Galois group of K over Q.
(a) Find the normal subgroups of G. (10%)
(b) Find the subfields of K which is normal over Q. (5%)