1. a. Show that $\langle p \rangle = 0$ for all state of a particle in a 1-D box with length $a$.

   b. What are the degeneracies of the first three energy levels of a particle in a 3-D box with $a = b = 1.5c$.

2. a. Determine the term symbol of the ground state electron configuration of $B_2$.

   b. Show that $\psi_{sp} = \dfrac{1}{\sqrt{2}}(2s \pm 2p_z)$ is normalized.

3. a. Integrate the rate equation $-\dfrac{dC}{dt} = kC^{1/2}$.

   b. What are the units of $k$.

   c. Derive the expression for $t_{1/2}$ in terms of $k$ and $C_0$.

4. Determine the normal modes of vibration for $C_{20}$ molecule and indicate the IR active and Raman active. (Hint: the $C_{20}$ has $I_h$ symmetry with the carbon is located in the center of equivalent triangle of icosahedron)

5. a. Calculate the entropy change if 2.0 mol of monatomic gas ($C_{p,m} = \dfrac{5}{2}R$) is heated from 300K to 400K and the pressure increases from 2.5 *atm* to 3.5 *atm*.

   b. Describe a Carnot engine being one mole of ideal gas, write the expression for the work done in each step.

## 每題二十分

11. The Icosahedral Groups*

| $I_h$ | $E$ | $12C_5$ | $12C_5^2$ | $20C_3$ | $15C_2$ | $i$ | $12S_{10}$ | $12S_{10}^3$ | $20S_6$ | $15\sigma$ | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $A_g$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | $x^2+y^2+z^2$ |
| $T_{1g}$ | 3 | $\frac{1}{2}(1+\sqrt{5})$ | $\frac{1}{2}(1-\sqrt{5})$ | 0 | $-1$ | 3 | $\frac{1}{2}(1-\sqrt{5})$ | $\frac{1}{2}(1+\sqrt{5})$ | 0 | $-1$ | $(R_x,R_y,R_z)$ | |
| $T_{2g}$ | 3 | $\frac{1}{2}(1-\sqrt{5})$ | $\frac{1}{2}(1+\sqrt{5})$ | 0 | $-1$ | 3 | $\frac{1}{2}(1+\sqrt{5})$ | $\frac{1}{2}(1-\sqrt{5})$ | 0 | $-1$ | | |
| $G_g$ | 4 | $-1$ | $-1$ | 1 | 0 | 4 | $-1$ | $-1$ | 1 | 0 | | |
| $H_g$ | 5 | 0 | 0 | $-1$ | 1 | 5 | 0 | 0 | $-1$ | 1 | | $(2z^2-x^2-y^2,$ $x^2-y^2,$ $xy, yz, zx)$ |
| $A_u$ | 1 | 1 | 1 | 1 | 1 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | | |
| $T_{1u}$ | 3 | $\frac{1}{2}(1+\sqrt{5})$ | $\frac{1}{2}(1-\sqrt{5})$ | 0 | $-1$ | $-3$ | $-\frac{1}{2}(1-\sqrt{5})$ | $-\frac{1}{2}(1+\sqrt{5})$ | 0 | 1 | $(x,y,z)$ | |
| $T_{2u}$ | 3 | $\frac{1}{2}(1-\sqrt{5})$ | $\frac{1}{2}(1+\sqrt{5})$ | 0 | $-1$ | $-3$ | $-\frac{1}{2}(1+\sqrt{5})$ | $-\frac{1}{2}(1-\sqrt{5})$ | 0 | 1 | | |
| $G_u$ | 4 | $-1$ | $-1$ | 1 | 0 | $-4$ | 1 | 1 | $-1$ | 0 | | |
| $H_u$ | 5 | 0 | 0 | $-1$ | 1 | $-5$ | 0 | 0 | 1 | $-1$ | | |

*For the pure rotation group $I$, the outlined section in the upper left is the character table; the $g$ subscripts should, of course, be dropped and $(x, y, z)$ assigned to the $T_1$ representation.