

淡江大學九十三年學年度碩士班招生考試試題

系別：資訊管理學系

科目：離散數學導論

准帶項目請打「○」否則打「×」
簡單型計算機 <input checked="" type="checkbox"/>

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Note1: 務必依序作答

Note2: 建議使用中文作答，其中術語使用英文。

Note3: 本試題中 \mathbb{R} 代表實數系， \mathbb{Z}^+ 代表正整數集合。

1. Let $A_1 = \{x \mid -1 \leq x \leq 1\}$, $A_2 = \{x \mid 1 \leq x \leq 3\}$, $A_3 = \{x \mid -3 \leq x \leq -1\}$. Let the functions $f_1: A_1 \rightarrow \mathbb{R}$, $f_2: A_2 \rightarrow \mathbb{R}$ and $f_3: A_3 \rightarrow \mathbb{R}$ be defined by $f_1(x) = x^2$, $f_2(x) = x^2$, $f_3(x) = x^2$.

- (a) What are the images of the functions f_1, f_2 and f_3 ? [10%]
- (b) For each function, state whether it is one-to-one or not? [10%]
- (c) Proof (or disprove) your answers in (b) for f_1 and f_2 . [10%]

2. Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} -x & \text{for } x > 0 \\ x^2 & \text{for } x \leq 0 \end{cases}$$

- (a) Find the inverse function f^{-1} . [10%]
- (b) Find the composition function $f \circ f$. [10%]

3. For any function $f: \mathbb{Z}^+ \rightarrow \mathbb{R}$, define its difference function $\Delta f: \mathbb{Z}^+ \rightarrow \mathbb{R}$ by

$$\Delta f(n) = f(n+1) - f(n).$$

- (a) If $f(n) = 2n^3 + 2n$, find the functions $f_1 = \Delta f$ and $f_2 = \Delta f_1$. [10%]
- (b) If $f(n) = 4n + 3$, find the function h such that $f = \Delta h$ and $h(1) = 2$. [10%]

4. Let $r = \sqrt{5}$, $a = (1+r)/2$, $b = (1-r)/2$. Define $S_n = (a^n + b^n)$ for $n = 1, 2, \dots$

- (a) Find S_1, S_2, S_3, S_4, S_5 . [10%]
- (b) Proof by induction that S_n is an integer for each n . [10%]

- 5. (a) Explain what is a directed acyclic graph. [5%]
- (b) Explain what is a leaf in a tree. [5%]