

淡江大學九十四學年度碩士班招生考試試題 ¹¹⁰⁻

系別：財務金融學系

科目：統計學

准帶項目請打「V」	
	簡單型計算機
本試題共 頁	

1. Assume that you are given the following joint density function,

$$f(x, y) = \begin{cases} 8xy, & 0 \leq x \leq 1, 0 \leq y \leq x \\ 0, & \text{otherwise} \end{cases}$$

- A. (10%) Please find $E(X)$.
- B. (10%) Please find $V(X)$.

2. Let X and Y be random variables, let μ_X (μ_Y) and σ_X^2 (σ_Y^2) be the mean and variance of X (Y), and let σ_{XY} be the covariance between X and Y .

- A. (10%) Show that $E(Y^2) = \sigma_Y^2 + \mu_Y^2$.
- B. (10%) Show that $E(XY) = \sigma_{XY} + \mu_X\mu_Y$.
- C. (15%) Show that $\sigma_{XY}^2 \leq \sigma_X^2\sigma_Y^2$. [Hint: One possible way to prove this needs to use $V(aX + bY) = a^2\sigma_X^2 + 2ab\sigma_{XY} + b^2\sigma_Y^2$.]

3. Suppose that you have the following estimated result,

$$\widehat{\text{Gain}} = 11.101 + 1.433 \text{Tuition}$$

where "Gain" is the gain in salary and "Tuition" is the annual tuition, both in thousands of dollars.

- A. (10%) What does 1.433 (the coefficient of "Tuition") mean?

Now, suppose salary gain is measured in actual dollars (Gain^*) and tuition is defined in hundreds of dollars (Tuition^*).

- B. (15%) Please show how to obtain the values of $\hat{\alpha}$ and $\hat{\beta}$ in the following regression.

$$\widehat{\text{Gain}^*} = \hat{\alpha} + \hat{\beta} \text{Tuition}^*$$

4. The density for each observation $y_i, i = 1, 2, \dots, n$ is,

$$f(y_i|\theta) = \frac{e^{-\theta}\theta^{y_i}}{y_i!}$$

- A. (10%) Write down the log likelihood function.
- B. (10%) Consider a random sample 5, 0, 1, 1, 0, 3, 2, 3, 4, 1. Please use this data set to obtain the maximum likelihood estimate of θ .