淡江大學九十三學年度碩士班招生考試試題

系別：財務金融學系　　　　科目：統　計　學

准帶項目請打「○」否則打「×」
○ 簡單型計算機

本試題雙面印製

(一) 選擇題 (Multiple Choice): 5*4%=20%

(1) Suppose a certain mutual fund has an annual rate of return that is approximately normally distributed with mean (expected value) 5% and standard deviation 4%. What is the probability that your 1-year return will exceed 10%. (From Table of the Standard Normal Distribution, area under Z=2.5 is 0.4938 and Z=1.25 is 0.3944)

(a) 0.0062　(b) 0.1056　(c) 0.5062　(d) 0.6056

(2) For a simple regression model $y_t = \beta_1 + \beta_2 x + e_t$, the unbiased estimator of the error variance is:

(a) $\hat{\sigma}^2 = \frac{\sum \hat{e}_t^2}{T}$　(b) $\hat{\sigma}^2 = \frac{\sum \hat{e}_t^2}{T-1}$　(c) $\hat{\sigma}^2 = \frac{\sum \hat{e}_t^2}{T-2}$　(d) $\hat{\sigma}^2 = \frac{\sum \hat{e}_t^2}{T-3}$

(3) Let $X_1, X_2, \ldots, X_n$ be independent random variables that all have the same probability distribution, with mean β and variance σ^2. Since we know that $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then E[$\bar{X}$] and Var[$\bar{X}$] are the value of ____ and ____, respectively.

(a) β/n and σ^2/n　(b) β and σ^2/n　(c) β and σ^2　(d) β/n and σ^2

(4) The explained sum of squares (SSR=sum of squares regression) is the form of:

(a) $\sum(\hat{y}_t - \bar{y})^2$　(b) $\sum(y_t - \bar{y})^2$　(c) $\sum(y_t - \hat{y}_t)^2$　(d) $\sum \hat{e}_t^2$

(5) If an intercept is present in the regression model, then $\sum(\hat{y}_t - \bar{y})\hat{e}_t =$

(a) 0　(b) 1　(c) $\hat{e}_t$　(d) $\sum \hat{e}_t^2$

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(二) <u>推導及計算題:</u> 8*10=80%

(1) 某台商於大陸上海市浦東新區投資一筆生意、發現其投資額度與其營業獲
10% 利率呈現以下聯合機率分配關係:

獲利率(Y) / 投資額(X)	10%	20%	30%
10 萬元	0.1	0.2	0.1
20 萬元	0.1	0.1	0.1
30 萬元	0.1	0.1	0.1

試求: (a) E(Y)　(b) Var(X)　(c) Cov(X,Y)　(d) E(X/y=20%)
(e) Are X & Y independent?

10%
(2) Suppose that X is a random variable for which E(X)=1, $E(X^2)=4$, and $E(X^3)=10$.
Find the value of the third central moment of X ?

10%
(3) Suppose that $X_1,\ldots,X_n$ form a random sample from a distribution for which the p.d.f. $f(x/\lambda)$ is as follows:

$$f(x/\lambda)=\begin{cases}\lambda x^{\lambda-1} & for\ \ 0<x<1\\ 0 & otherwise.\end{cases}$$

Also, suppose that the value of λ is unknown ($\lambda>0$). Find the maximum likelihood estimator (MLE) of λ ?

10%
(4) The Suppose that weather can be only sunny or rainy (two states).
The Markov chain transition matrix is as follows:

	S	R
S	0.7	0.3
R	0.6	0.4

(a) As we observed, it is sunny today (Saturday). What is the probability that it will be sunny on both Tuesday and Wednesday.
(b) Suppose the probability that it will be sunny on tomorrow (Sunday) is 0.3.
Determine the probability that it will be rainy on Wednesday.

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10%

(5) For the case of the first order autocorrelation :

$$\begin{cases} y_t = \beta_1 + \beta_2 X_t + e_t \\ e_t = \rho e_{t-1} + v_t \qquad where \ \ -1 < \rho < 1 \ \ \& \ \ v_t \ \text{ is white noise} \end{cases}$$

Prove that $Var(e_t) = \sigma_e^2 = \dfrac{\sigma_v^2}{1-\rho^2}$

10%

(6) The observations of y and x are as following:

x	1	2	3	4	5	6
y	4	6	7	7	9	11

a) Use the formulas for the least squares estimates to computer the least squares estimates of the slope and the intercept.

b) Use the least squares estimates from (a), computer the least squares residuals $\hat{e}_t$. Find their sum.

10%

(7) In an estimated simple regression model, based on 24 observations, the estimated slope parameter is .310 and the estimated standard error is .082.

a) Test the hypothesis that the slope is zero, against the alternative that it is negative, at the 5% level of significance. Draw a sketch showing the rejection region. (Critical t-value: $t_c = -1.717$)

b) Test the hypothesis that the estimated slope is 0.5, against the alternative that it is not, at the 5% level of significance. (Critical t-value: $\pm t_c = \pm 2.074$)

10%

(8) Suppose that a simple linear regression has quantities $R^2 = 0.7911$, SST = 552.36 and T = 20. Find $\hat{\sigma}^2$