

淡江大學八十八學年度碩士班招生考試試題

系別：財務金融學系

科目：統計學

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本試題雙面印製

(每題均為十分，共十題)

1. 擲公正硬幣 2 次。令 $D_1 = 1$ ，第一次為人頭（正面）； $D_1 = 0$ ，其他情形。
 $D_2 = 1$ 第二次為人頭； $D_2 = 0$ ，其他情形。 $D_3 = 1$ 兩次結果相同； $D_3 = 0$ 兩次結果不相同。

- a) D_1, D_2 與 D_3 中任兩個配對是否獨立？
- b) D_1, D_2 與 D_3 三者是否獨立？

2. 生物學中隨時間比例成長的一般模型為 $E(Y|t) = 1 - \exp^{-\beta t}$ ，其中 Y 表組織中病變細胞比例， t 表時間。現有 n 個觀察值。如何估計並建立 β 的信賴區間？（必須說明假設條件）

3. Let $X \sim N(0, 1)$ and consider the transformation $Y = \exp^X$, find the mean and variance of Y .

4. Given $Y = 1 + 4X + Z$, X and Z are independent random variables with $E(X) = 3$, $\text{Var}(X) = 15$, $E(Z) = 0$, and $\text{Var}(Z) = 9$. Find
 a) $E(Y)$ b) $E(Y|X)$ c) $\text{Var}(Y)$ d) $\text{Var}(Y|X)$

5. The joint probability density function of two discrete random variables X and Y is given by the following table

		y		
		1	3	9
x		2	1/8	1/24
		4	1/4	1/4
		6	1/8	1/24
				1/12

- a) Find the covariance of X and Y .
- b) Are X and Y independent?

6. Let X_1, X_2, \dots, X_{10} denote a random sample of size 10 from a distribution that is $N(2, 9)$. If $W = 2X_1 - 4X_6 + X_{10}$ and $Z = X_2 + 3X_4 - 2X_8$.
 a) What are the distribution of W and Z ?
 b) Are W and Z independent?

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7. Suppose X and Y are continuous random variables with joint probability density $f(x, y) = 4(x - xy)$ if $0 < x < 1$ and $0 < y < 1$, and zero otherwise.

- a) Are X and Y independent?
b) Find $P[X < Y]$.

8. Let X_1, X_2, \dots, X_n be a random sample from a uniform distribution,

$X_i \sim \text{UNIF}[\theta - 1, \theta + 1]$. Is \bar{X} an unbiased estimator of θ ?

9. Suppose that a random variable $X \sim N(35, 400)$. In addition, suppose that you are

unaware that $E(X) = \mu = 35$, and that you wish to use a random sample of 42 observations to test $H_0: \mu = 30$ against $H_1: \mu > 30$

- a) If you use a 5% significance level, what is the probability of making a Type II error?
b) What is the probability of a Type I error?

10. Let X_1 and X_2 be a random sample from $N(0, 1)$.

- a) What is the distribution of $\frac{(X_1 + X_2)^2}{(X_2 - X_1)^2}$?

- b) What is the distribution of $\frac{X_2 + X_1}{X_1 - X_2}$?