淡江大學八十七學年度碩士班入學考試試題

系別： 財務金融學系　　科目： 統計學

本試題共 2 頁

1. [10%] A sample of ten readings of temperature in Fahrenheit degrees has a mean of 41 and a standard deviation of 9. Determine the mean and the standard deviation for these readings in terms of Celsius degree. (Note: $32^0C = 0^0F$ and $212^0F = 100^0C$).

2. [20%] 假設一簡單線性迴歸模型 $Y_i = \alpha + \beta X_i + \varepsilon_i$，其中誤差項 ε_i 爲獨立隨機變數 $\varepsilon \overset{iid}{\sim} N(0, \sigma^2)$. $\underset{\hat{\alpha},\hat{\beta}}{Min} \sum_{i=1}^{n} \left(Y_i - \hat{\alpha} + \hat{\beta}X\right)^2$ 定義 $e_i = Y_i - \hat{\alpha} - \hat{\beta}X_i$. 求下列各小題之值？ (此一簡單線性迴歸模型，符合所有線性迴歸模型之基本假設)

a) $E\left(\sum_{i=1}^{n} e_i^2\right)$?　　b) $Var\left(\sum_{i=1}^{n} e_i^2\right)$?

c) $E\left(\sum_{i=1}^{n} X_i e_i\right)$?　　d) $Var\left(\sum_{i=1}^{n} X_i e_i\right)$?

3. [10%] 大陸一胎化政策產生許多社會問題，如：可能產生女男人數不相等的問題。因此有學者提出較爲符合中國人傳宗接代觀念的人口政策，也就是只要是生女兒不限制生小孩的人數，直到生男孩爲止。學者指出此政策不會造成女男人數不相等。你是否同意上述的說法？請問下一代的人口總數與上一代人口總數的變化？（假設生男與生女的機率均爲 0.5）
[詳述求得答案的過程及假設]

4. [10%] 一項調查結果發現在 1000 位家長中，有 550 位家長不贊成體罰。此結果是否表示大多數的家長不贊成體罰？ (顯著水準 $\alpha = 0.05$)

5. [20%] Consider a sample of identically and independently distributed variables $X_1, X_2, \ldots\ldots, X_n$ each having a mean μ and variance σ^2. Find each of the following: (for $i = 1$ to n, $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$, and $i \neq j$)

a) $E(X_i \overline{X})$　　b) $E\left((X_i (X_j \overline{X})\right)$

c) $Var(X_i - \overline{X})$　　d) $Cov\left[(X_i - \overline{X}), \overline{X}\right]$

淡江大學八十七學年度碩士班入學考試試題

系別： 財務金融學系　　科目： 統計學

6. [10%] The joint distribution of X and Y is as follows.

		X				
		-2	-1	0	1	2
Y	10	0.09	0.15	0.27	0.25	0.04
	20	0.01	0.05	0.08	0.05	0.01

a) Find the marginal distributions of X and Y.

b) Are X and Y correlated?

c) Are X and Y independent?

7. [10%] When testing a null hypothesis $H_0 : \mu = 60$ against an alternative $H_A : \mu > 60$, the test statistic

$TS = \dfrac{\overline{X} - 60}{\sigma/\sqrt{n}}$ is used. The following test criterion is adopted:

Reject H_0 if $TS > 2.1$ but accept H_0 if $TS < 2.1$

If the test statistic has a standard normal distribution under H_0, calculate the approximate probability of a **type I error**. Given $n = 100$ and $\sigma = 12$, calculate the approximate probability of a **type II error** when $\mu = 75$.

8. [10%] Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with a common mean μ and variance σ^2. Two estimators of the mean are propose

$$\hat{\mu}_1 = \frac{X_1 + X_n}{2},$$

$$\hat{\mu}_2 = \frac{\sum_{i=1}^{n} X_i}{n}.$$

Compare the properties of these estimators.