1. Two random variables X and Y have the following means, variances, and covariance:

$$E(X) = 5, \; Var(Y) = 10, \; E(Y) = 2, \; Var(Y) = 20, \; Cov(X, Y) = 5$$

   If $A = 2X+Y$, and $B = X-2Y$, find the means and variances of A and B. Show that

   $$E(AB) = 2E(X^2) - 2E(Y^2) - 3E(XY).$$ Hence find the covariance between A and B.

2. Let $X_1, X_2, ..., X_n$ be a random sample drawn from a population with mean μ and variance σ^2. Let $\overline{X} = \dfrac{1}{n}\sum_{i=1}^{n} X_i$ be the sample mean. Show that

   $$E(\overline{X}) = \mu \quad \text{and} \quad Var(\overline{X}) = \frac{\sigma^2}{n}.$$ Next let $Y = \dfrac{1}{n}\sum a_i X_i$ where the a_i's are fixed constants. Derived $E(Y)$ and $Var(Y)$.

3. The following data refers to the demand for apples, Y, in kg, and the price of apples, X, in pence per kg, on 10 different market stalls:

X	99	91	70	79	60	55	70	101	81	67
Y	22	24	23	26	27	24	25	23	22	26

   (1) $\sum y^2 = \sum (Y - \overline{Y})^2 = ?$　$\sum x^2 = \sum (X - \overline{X})^2 = ?$　$\sum xy = ?$

   (2) Assuming $Y = \alpha + \beta X + \varepsilon$, obtain the OLS estimators of α and β, and their estimated standard errors.

   (3) Obtain R^2, interpret your results.

4. Suppose the regression equation $Y_t = \beta_1 + \beta_2 X_t + \varepsilon$ is to be estimated, but the disturbance follows a first-order autoregressive process
   $$\varepsilon_t = \rho_1 + \rho\varepsilon_{t-1} + u_t$$
   (1) Find $E(\varepsilon)$? and $Var(\varepsilon)$?
   (2) Are OLS estimators BLUE? Why?
   (3) Explain how would you estimate this equation?