淡江大學九十學年度碩士班招生考試試題

系別：統計學系　　　　　　科目：統　　計　　學

本試題共　/　頁

※計算請詳述計算過程、證明請詳列證明步驟，否則不予計分※

1. 試回答下列分配之平均數 E(X)(每個各兩分)及變異數 Var(X)，請寫清楚題號，
   不需附計算過程：

   (a) 若隨機變數 X 服從試驗次數為 n，成功機率為 p 之二項分配。(5%)

   (b) 若隨機變數 X 之機率密度函數為 $f(x) = \dfrac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\lambda x), x > 0$。(5%)

   (c) 若隨機變數 X 之機率密度函數為 $f(x) = \dfrac{e^{-\lambda} \lambda^{x}}{x!}, x = 0,1,2,3,\cdots$。(5%)

   (d) 若隨機變數 X 之機率密度函數為 $f(x) = \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, 0 < x < 1$。
   (5%)

2. If you want to be 95% confident of estimating the population proportion to within
   an error of ± 0.04, what sample size is needed ? (15%)(查表值自行計算，不另附
   統計表)

3.
   (a) 試敘述中央極限定理(5%)
   (b) 使用特徵函數(characteristic function)證明中央極限定理(15%)

3. Let $X_1, X_2, \cdots, X_n$, $n > 2$, be a random sample from the binomial distribution
   $B(1, \theta)$.
   (a) Show that $Y_1 = X_1 + X_2 + \cdots + X_n$ is a complete sufficient statistic for θ.
   (10%)
   (b) Find the function $\varphi(Y_1)$ which is the unbiased minimum variance estimator
   of θ. (5%)
   (c) Let $Y_2 = (X_1 + X_2)/2$ and compute $E(Y_2 \mid Y_1 = y_1)$. (10%)

4. If $X_1, X_2, \cdots, X_n$ is a random sample from a distribution with pdf
   $f(x) = \frac{1}{2}\theta^3 x^2 \exp(-\theta x), x > 0$, $\theta > 0$.

   (a) Find the m.l.e., $\hat{\theta}$ of θ. Is $\hat{\theta}$ unbiased ? (10%)

   (b) Show that $\dfrac{X_1}{Y}$ and Y are independent, where $Y = \sum_{j=1}^{n} X_j$ (10%).
   (Hint: First find the pdf of Y)