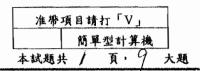
淡江大學 98 學年度碩士班招生考試試題

系別:統計學系

科目:基礎數學(含微積分、線性代數)



1. Show that for x > 0, $\left(\int_{x}^{\infty} t e^{-t^2/2} dt\right)^2 \le \int_{x}^{\infty} e^{-t^2/2} dt \times \int_{x}^{\infty} t^2 e^{-t^2/2} dt$. (Hint: consider the inequality $\int_{x}^{\infty} (at+1)^2 e^{-t^2/2} dt \ge 0$, where a is any real number.) (10%)

2. Evaluate the following limits

a)
$$\lim_{h \to 0} \frac{(2+h)^5 - 2^5}{h}$$
. (6%)

b)
$$\lim_{x\to 3} \frac{x^2-x-6}{x-3}$$
. (5%)

c)
$$\lim_{n\to\infty} s_n$$
, where $s_n = \sum_{k=1}^n \frac{1}{k(k+1)}$. (8%)

- 3. Let the sequence $\{a_n\}_{n=1}^{\infty}$ be defined as $a_1 = 1$ and $a_{n+1} = \sqrt{2 + a_n}$, $n \ge 1$. Evaluate $\lim_{n \to \infty} a_n$. (10%)
- 4. a) Prove that $\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$, provided that the two integrals exist. (Hint: use change-of-variable to rewrite the integral by letting $u = \pi x$.) (6%)
 - b) Use 4a) to evaluate the integral $\int_0^x x(\cos x)^4 \sin x \, dx$. (7%)
- 5. Let $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$ be a collection of the n points on the x-y plane. Suppose that $\sum_{i=1}^{n} x_i = 0$ and $x_i \neq 0$, for some $i = 1, 2, \dots, n$. Prove that the quantity $S(a, b) = \sum_{i=1}^{n} (y_i ax_i b)^2$ is minimized, when $a = \left(\sum_{i=1}^{n} x_i^2\right)^{-1} \sum_{i=1}^{n} x_i y_i$, $b = n^{-1} \sum_{i=1}^{n} y_i$. Justify your answer by second derivative test. (12%)
- 6. Find the value of t for which the following system is consistent and solve the system for this value of t. (5%)

$$\begin{cases} x+y=1\\ tx+y=t\\ (1+t)x+2y=3 \end{cases}$$

- 7. Let C be an invertible matrix, show that $(C^{-1})^t = (C^t)^{-1}$. (5%)
- 8. Let $A = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$, $B = (I A)(I + A)^{-1}$ and $C = (I B^3)(I + B^3)^{-1}$, where I is the 2×2 identity matrix.
 - a) Show that $(I+A)^{-1} = (I+A^{-1})/\det(I+A)$ and hence show that B is skew-symmetric (i.e. B' = -B). (Hint: A is orthogonal, so $A' = A^{-1}$.) (8%)
 - b) It is known that $(I + B^3)$ and $(I B^3)$ are both invertible. Use 7) and 8a) to show that C is orthogonal (i.e. CC' = I). (8%)
- 9. Let $A = \begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix}$, use the fact $A^2 = 4A 3I_2$ and mathematical induction, to prove that $A^n = \frac{(3^n 1)}{2}A + \frac{(3 3^n)}{2}I_2, \text{ for } n \ge 1,$

where I_2 is the 2×2 identity matrix.