淡江大學97學年度碩士班招生考試試題

系別：統計學系　　　　科目：基礎數學(含微積分、線性代數)

准帶項目請打「V」	
V	簡單型計算機

本試題共　頁，　大題

下列所有考題皆為計算或證明題，皆須附帶寫出計算或證明過程，否則不予計分。

1. Determine convergence or divergence of the following two series (State briefly the theorems or methods of test you employ in your work)

   a) $\sum_{k=1}^{\infty}\frac{\sin k}{k^2}$ (5%)　　b) $\sum_{n=1}^{\infty}(-1)^n a_n$ $\left(其中\ a_n=\int_2^{2+2/n}\frac{1}{x}dx\right)$ (5%)

2. Evaluate the given limits, if possible

   a) $\lim_{x\to 0}\frac{x-\int_0^x \cos t\ dt}{x-\int_0^x e^{t^2}dt}$ (6%)　　b) $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}\sqrt{n+1}}+\frac{1}{\sqrt{n}\sqrt{n+2}}+\cdots+\frac{1}{\sqrt{n}\sqrt{n+n}}\right)$ (10%)

3. Use Mean Value Theorem to prove that: $\frac{x-1}{x}<\ln x<x-1$, whenever $x>1$. (8%)
4. Prove that the function $f(x)=x^3+3x+1$ has exactly one real root. (7%)
5. a) Write the Maclaurin series of the function $f(x)=e^x$ and its radius of convergence. (5%) b) Propose a strategy to approximate the definite integral $\int_0^1 e^{x^2}dx$ (just state the approximation strategy, don't evaluate it). (5%)
6. Let $A=[a_{ij}]_{n\times n}$, $a_{ij}\in R$ be an $n\times n$ matrix, prove that: the system $A_{n\times n}\cdot X_{n\times 1}=0_{n\times 1}$ has non-trivial solution (i.e. the system has solution X that is not zero vector) $\Leftrightarrow$ (if and only if) $\det(A)=0$. (10%)
7. Let $A=\begin{pmatrix}2 & t^2 & 5\\ 0 & t+1 & t\\ 0 & t+5 & 2t+5\end{pmatrix}$, $t\in R$. a) For $t=0$, use Cramer's rule to calculate A^{-1}. (8%) b) For all $t\in R$, prove that A^{-1} always exists. (5%)
8. Let $T:R^3\to R^4$ be a linear transformation, where for 3×1 vector X, $T(X)=\begin{pmatrix}1&0&0\\0&1&1\\1&1&0\\0&0&1\end{pmatrix}X$. Find the bases respectively for a) the kernel (null space) of T and b) the image of T. (10%)
9. Let $A=\begin{pmatrix}3/5 & 4/5\\ 2/5 & 1/5\end{pmatrix}$. a) Find the eigenvalues of A and their corresponding eigenvectors (6%); b) prove that when $n\in N$, $A^n\to\begin{pmatrix}2/3 & 2/3\\ 1/3 & 1/3\end{pmatrix}$, as $n\to\infty$ (where $A^2=A\times A$, and $A^n=A^{n-1}\times A$ for $n=3,4,5,\cdots$). (10%)