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## 淡江大學 96 學年度碩士班招生考試試題

系別：統計學系

科目：基礎數學(含微積分、線性代數)

准帶項目請打「V」

簡單型計算機

本試題共 2 頁

本試題雙面印製

1) (a) Let  $f\left(\frac{x-1}{2x}\right) = e^{\frac{3}{2}x}$ . Prove that  $f'(0) = 3f(0)$ . (4%)

(b) Let  $x f(x^2) = \ln(x^4 + 1)$ . Find  $f'(1)$  and  $f''(1)$ . (8%)

2) Let  $f$  be a continuous differentiable function defined on the set  $R$  of all real numbers and  $F = f - 1$ .

(a) Prove that  $\min_{a \leq x \leq b} f'(x) \leq \frac{F(b) - F(a)}{b - a} \leq \max_{a \leq x \leq b} f'(x)$  for any  $a < b$ . (5%)

(b) If  $F(1) = 0$ ,  $F' < 0$  on  $(-\infty, 1)$  and  $F' > 0$  on  $(1, \infty)$ , prove that  $f'(1) = 0$  and  $f(x) \geq 1$ , for all  $x \in R$ . (5%)

3) (a) If  $\lim_{x \rightarrow 4} \frac{2f(x) - 3}{x - 1} = 1$ , find  $\lim_{x \rightarrow 4} f(x)$ . (4%)

(b) Find  $\lim_{x \rightarrow \sqrt{3}} \frac{\sqrt{x^2 + 1} - 2}{\frac{1}{3}x^2 - 1}$ . (4%)

(c) Find  $\lim_{x \rightarrow 1^+} \frac{1}{\ln x^x} \int_1^{x^2} e^{\frac{t-1}{2}} dt$  (4%)

4) Evaluate the following integrals:

(a)  $\int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2}} dx$ . (8%)

(b)  $\iint_A x^2 y dx dy$ , where  $A = \{(x, y) | 1 < 2x < y < 2\}$ . (8%)

5) Let  $A$  be a  $4 \times 4$  matrix such that  $A^2 + 3A - 4I = 0$ , where  $I$  is the identity matrix. If  $\det(A - I) \neq 0$ , find  $A$  and its inverse. Moreover, find  $\det(A - I)$ . (8%)

6)  $n$  functions are said to be functionally independent if no one of them is a function of the others. Let  $f_1(x) = \frac{1}{x}$ ,  $f_2(x) = \frac{1}{2x^2}$ , and  $f_3(x) = \frac{1}{x^3}$ .

(a) Prove or disprove that  $f_1, f_2, f_3$  are linearly independent. (8%)

(b) Prove or disprove that  $f_1, f_2, f_3$  are functionally independent. (4%)

◀ 注意背面尚有試題 ▶

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$$7) \text{ Let } A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}.$$

- (a) Find the eigenvalues and the eigenspaces for  $A$ . (12%)  
 (b) Find the algebraic and geometric multiplicities of each eigenvalue.  
 Is  $A$  diagonalizable? Why? (4%)  
 (c) Calculate  $A^6$ . (5%)

8) Let  $T: R^5 \rightarrow R^4$  be a linear transformation defined by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{bmatrix} x_1 + 3x_2 + 4x_3 + 7x_5 \\ 3x_1 - 2x_2 + x_3 - x_5 \\ -x_1 - x_3 - x_5 \\ x_1 + 2x_2 + 5x_3 + x_4 + 6x_5 \end{bmatrix}$$

- (a) Find a basis for the range of  $T$ . (5%)  
 (b) Find the nullity of  $T$ . (4%)