

淡江大學九十四學年度碩士班招生考試試題 ¹⁴⁵⁻¹

系別：統計學系

科目：基礎數學(含微積分、線性代數)

准帶項目請打「V」

簡單型計算機

本試題共 頁

1. (12 pts) Find the following limits:

$$(a) \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n \quad (b) \lim_{x \rightarrow \infty} x \log \left(\frac{x+1}{x-1}\right) \quad (c) \lim_{x \rightarrow 1} \frac{x + x^2 + \cdots + x^n - n}{x-1}$$

2. (8 pts) Find the derivatives of the following functions:

$$(a) f(x) = \frac{e^{3x}}{x+1}$$

$$(b) \text{ Let } f\left(\frac{x-1}{x+1}\right) = x. \text{ Find } f'(0).$$

3. (14 pts) Consider the gamma function $\Gamma(a) = \int_0^{\infty} x^{a-1} e^{-x} dx$. Show that

$$(a) \Gamma(a+1) = a\Gamma(a) \quad (b) \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

4. (16 pts) Find the following double integral:

$$(a) \iint_B xy \, dx dy, \text{ where } B = \{(x, y) \mid x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}.$$

$$(b) \iint_A e^{-y} \, dy dx, \text{ where } A = \{(x, y) \mid 0 < x < y < \infty, x + y > 1\}$$

5. (10 pts) Let A and B be $n \times n$ matrices, and let C be a nonsingular $n \times n$ matrix. Suppose that $C^{-1}AC = B$.

$$(a) \text{ Prove that } \det(A) = \det(B).$$

(b) Show that A and B have the same eigenvalues.

6. (10 pts) If the $n \times n$ symmetric matrix A has the property $x^T Ax > 0$ for all possible $n \times 1$ vector x except $x = 0$, then A is said to be a positive definite matrix.

$$(a) \text{ Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \text{ where } |x_1| + |x_2| > 0. \text{ Show that } A \text{ is a positive definite matrix.}$$

(b) Let P be a nonsingular matrix and A be a positive definite matrix. Show that $P^T A P$ is positive definite.

$$7. (20 \text{ pts}) \text{ Let } A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}.$$

(a) Find the eigenvalues of A and the corresponding eigenvectors.

(b) Find the eigenvalues of A^5 .

(c) Show that A is diagonalizable.

(d) Compute A^5 .

8. (10 pts) Consider the following linear system $Ax = b$ given by

$$\begin{aligned} x_1 - x_2 &= 4 \\ 3x_1 + 2x_2 &= 1 \\ -2x_1 + 4x_2 &= 3 \end{aligned}$$

Find the orthogonal projection of b on the column space of A .