淡江大學九十三學年度碩士班招生考試試題

系別：統計學系　　　　科目：基礎數學(含微積分、線性代數)

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(1) Find the following limits: (12%)

(a) $\lim_{x\to\infty}\dfrac{\int_0^x e^{t^2}\,dt}{e^{x^2}}$　　(b) $\lim_{x\to 0^+}\sqrt{x}\log\log\dfrac{1}{x}$　　(c) $\lim_{x\to -1}\dfrac{e^{x+1}-1}{x^3+1}$

(2) Find the derivatives of the following functions f *and* g: (10%)

(a) $f(x)=\left(\dfrac{\sqrt{x}}{1+x}\right)^2$　　(b) $xg(x)+2x+3g(x)=1$

(3) (a) State the Mean Value Theorem in Calculus. (6%)

(b) Suppose that f is a function that is continuous on $[1,3]$ and differentiable on $(1,3)$. Prove that f is increasing on $[1,3]$, if the derivative $f'>0$ on $(1,3)$, and f is decreasing on $[1,3]$, if the derivative $f'<0$ on $(1,3)$. (6%)

(4) (a) Find the following double integral:

$\iint_A (x^2y+x-1)\,dx\,dy$, where $A=\{(x,y)\in R^2 \mid 0<x^2<y<1\}$ (8%)

(b) Suppose that $\iint_A c\,x\,dxdy=1$, where $A=\{(x,y)\in R^2 \mid x>0, y>0, 1\le x+y<2\}$

Find the constant c. (8%)

(5) Solve the following matrix equation for X: (9%)

$$\begin{bmatrix}1&4&2\\0&2&1\\3&5&3\end{bmatrix}X=\begin{bmatrix}2&-1&5&7&8\\4&0&-3&0&1\\3&5&-7&2&1\end{bmatrix}$$

(6) Let $A=\begin{bmatrix}1&4&5&2\\2&1&3&0\\-1&3&2&2\end{bmatrix}$.

(a) Find the corresponding reduced row-echelon form matrix B of A. (6%)

(b) Find a basis for the null space of A. (4%)

(c) What is the rank of A? (3%)

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(7) Let $A=\begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix}$.

(a) Find the eigenvalues and the eigenspaces for A. (12%)

(b) What are the algebraic and geometric multiplicities of each eigenvalue? (3%)

(c) Is A defective? Is A diagonalizable? Why? (4%)

(8) (a) Let $T: R^n \to R^m$ be a linear transformation, and let $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_n$ be the unit vectors in R^n.

Prove that $T(\vec{x}) = A\vec{x}, \forall \vec{x} \in R^n$, where $A = [T(\vec{e}_1), T(\vec{e}_2), \ldots, T(\vec{e}_n)]$. (5%)

(b) Let $T: R^3 \to R^2$ be a linear transformation defined by

$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -x_1 + 2x_2 - 5x_3 \\ 2x_1 + 3x_2 + x_3 \end{bmatrix}$. Find a matrix A such that

$T(\vec{x}) = A\vec{x}, \forall \vec{x} \in R^3$. (4%)