淡江大學九十二學年度碩士班招生考試試題

系別：統計學系　　科目：基礎數學（含微積分、線性代數）

准帶項目請打「○」否則打「×」
簡單型計算機
×

1. (a)求解 $\lim_{\lambda\to 0}\dfrac{\left(1+\dfrac{a\lambda}{x}\right)^x-\left(1+\dfrac{b\lambda}{x}\right)^x}{\lambda}$ (5%) (b) 求解 $\lim_{x\to 0}\dfrac{a^x-1}{x}$，$a>0$ (5%)

2. 若 $y=x^2+1$，$u=\sqrt{x^2+1}$，試證明 $\dfrac{dy}{du}=2\sqrt{y}$ (5%)

3. 設 $f'(a)$ 存在，求解 $\lim_{x\to a}\dfrac{xf(a)-af(x)}{x-a}$ (10%)

4. 已知積分均值定理爲 $f(c)=\dfrac{1}{b-a}\int_a^b f(x)dx$，設 $f(x)=2x^2+3x+3$，試求於區間[1,4]內之平均值 $f(c)$ (5%)

5. 試用極座標求解 $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ (10%)

6. (a)求解 $\sum_{n=1}^{\infty}\dfrac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n^2+n}}$ (5%)

(b)設 $S_n=\dfrac{n}{2n+1}$ 爲一級數的部份和，求該級數，並判斷其是否收斂 (5%)

7. Given the nullity and the rank of A, where $A=\begin{bmatrix}1&2&1\\-1&0&3\\1&5&7\end{bmatrix}$ (10%)

8. Use the Gram-Schmidt process to generate an orthonormal set from the given linearly independent vectors $\begin{bmatrix}1\\0\\1\\2\end{bmatrix},\begin{bmatrix}2\\1\\0\\2\end{bmatrix},\begin{bmatrix}1\\-1\\0\\1\end{bmatrix}$ (10%)

9. Evaluate $\det(A)$, where $A=\begin{bmatrix}1&2&-1&1\\-1&0&2&-2\\3&-1&1&1\\2&0&-1&2\end{bmatrix}$ (10%)

10. Use Cramer's Rule to solve

$$\begin{aligned}7x_1-2x_2&=3\\3x_1+x_2&=5\end{aligned}$$ (10%)

11. Find a matrix P that orthogonally diagonalizes A, and determine $P^{-1}AP$, where $A=\begin{bmatrix}3&1\\1&3\end{bmatrix}$. (10%)