淡江大學九十一學年度碩士班招生考試試題

系別：統計學系　　科目：基礎數學（含微積分、線性代數）　　136-1

准帶項目請打「○」否則打「×」	
計算機	字典
X	X

本試題雙面印製

1) Let $f(x)=\log x$.

(a) Show that $f'(1)=\lim_{h\to 0}\log(1+h)^{\frac{1}{h}}$. (4%)

(b) Show that $\frac{f(x+h)-f(x)}{h}=\frac{1}{x}\frac{f(1+t)-f(1)}{t}$, *where* $t=\frac{h}{x}$. (4%)

(c) Show that $f'(x)=\frac{f'(1)}{x}$. (4%)

2) Find the following limits: (12%)

(a) $\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{n}$.

(b) $\lim_{n\to\infty}\left(1+\frac{3}{n}+\frac{1}{n^2}\right)^{\frac{n}{2}}$.

(c) $\lim_{x\to\infty}\left(\frac{x}{x+2}\right)^{\frac{x}{2}}$.

3) (a) Let g,h be two differentiable functions and f be an integrable function

Find $\frac{d}{dx}\int_{g(x)}^{h(x)} e^{-2x}f(t)\,dt$. (8%)

(b) Find $\frac{d}{dx}\int_{2}^{\frac{1}{e^x}} \ln(1+e^{-2t})\,dt$. (6%)

4) (a) Find the following integral:

$$\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} xyz\,dz\,dy\,dx.$$ (8%)

(b) Let $f(x_1,x_2)=\begin{cases}2x_1x_2, 0<x_1<x_2<1\\ 0, \textit{otherwise}\end{cases}$

Find $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1,x_2)\,dx_1\,dx_2$. (6%)

◀注意背面尚有試題▶

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136-2

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X	X

5) Find a diagonal matrix A that satisfies (16%)

(a) $A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

(b) $A^{-3} = \begin{bmatrix} 27 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

In each case, find the inverse of A.

6) Let $T: R^3 \to R^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}, T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -17 \end{bmatrix}, T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 8 \end{bmatrix}.$$

(a) Find a matrix A such that $T(\vec{x}) = A\vec{x}, \forall\, \vec{x} \in R^3$. (4%)

(b) Is T invertible? If the answer is positive, find the corresponding inverse linear transformation. (6%)

(c) Find the eigenvalues of T. (10%)

7) Let $\vec{v}_1 = \begin{bmatrix} \lambda \\ -0.5 \\ 0.5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -0.5 \\ \lambda \\ -0.5 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -0.5 \\ -0.5 \\ \lambda \end{bmatrix} \in R^3$. Find all values of λ such that $\vec{v}_1, \vec{v}_2, and\ \vec{v}_3$ are linearly independent. (12%)