淡江大學九十學年度碩士班招生考試試題

系別：統計學系　　科目：基 礎 數 學（含微積分、線性代數）

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本試題共 1 頁

1. a) State the mean value theorem.

b) If f be a differentiable function with domain $[a,b]$. Suppose $m \le f'(x) \le M, \forall x \in [a,b]$. Prove that $f(a)+m(b-a) \le f(x) \le f(a)+M(b-a)$. <15%>

2. Find the following integrals:

a) $\int x^3 \cos(x^4+2)dx$ b) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx$ c) $\int \frac{1}{x\sqrt{x^6-4}}dx$ d) $\int \frac{\cos 3x}{\sin^2 3x}dx$

e) $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{1+y^3}\,dy\,dx$ <25%>

3.

a) If $f(x) = \int_{2x}^{x^3-4} \frac{x}{1+\sqrt{t}}dt$, find $f'(2)$.

b) Find $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta^2}$

c) Let $z = \mu^2 + \mu\nu - \nu^2$ and $\mu = e^{2x+y}, \nu = \ln\frac{y}{x}$. Find $\frac{\partial z}{\partial x}$. <20%>

4. Let $A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$

a) Find eigen values of A.

b) Find a matrix P such that $P'AP$ is diagonalized. <20%>

5. Let $L: R^3 \to R^3, L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \mu \\ \nu \\ w \end{bmatrix}$ be function defined by

$$\mu = x + y + z$$
$$\nu = 2y + 3z$$
$$w = 5x + 5y + z$$

a) Find matrix A such that $L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

b) Prove that L is invertable. Find the explicit equation for the corresponding inverse function L^{-1}.

c) Compute the Jacobian J for L and Jacobian j for L^{-1} and show that Jj=1. <20%>