淡江大學八十九學年度碩士班招生考試試題

系別：統計學系　　科目：基礎數學（含微積分、線性代數）

1) Find the following limits: (15%)

(a) $\lim_{x\to 3^+} \dfrac{x-\sqrt{3x}}{27-x^3}$　(b) $\lim_{x\to\infty}\left(\dfrac{x}{x+1}\right)^x$　(c) $\lim_{x\to 0^+} x^3 e^{1/x}$

2) Compute the following derivative and partial derivative: (12%)

(a) $\dfrac{d}{dx}\int_{x^2}^{2\ln x}\sqrt{1+3t^3}\,dt$

(b) $\dfrac{\partial^2}{\partial x\,\partial y} f(x^2+e^{2x}y-y^2,\ x+y^2)$.

3) Find the following integrals: (12%)

(a) $\int_0^1 x^3 e^{x^2}\,dx$

(b) $\iint_A xye^x\,dx\,dy$, where $A=\{(x,y)\mid 0<x<2y<1\}$

4) Let $f(t)=e^{\frac{t^2}{2}}$, $-\infty<t<\infty$.

(a) Find the MacLaurin's series of $f(t)$. (8%)

(b) Prove that $f^{(2k)}(0)=\dfrac{d^{2k}}{dt^{2k}}f(t)\Big|_{t=0}=\dfrac{(2k)!}{2^k\,k!}$, $k=1,2,\ldots$ (4%)

5) Let $S=\{\vec{w}_1,\vec{w}_2,\vec{w}_3,\vec{w}_4\}$ be a subset of vectors of R^3 and $W=Sp(S)$, the subspace spanned by S, where $\vec{w}_1=\begin{bmatrix}1\\2\\1\end{bmatrix}$, $\vec{w}_2=\begin{bmatrix}2\\5\\0\end{bmatrix}$, $\vec{w}_3=\begin{bmatrix}3\\7\\1\end{bmatrix}$, $\vec{w}_4=\begin{bmatrix}1\\1\\3\end{bmatrix}$.

(a) Prove or disprove that S is a linearly independent set. (6%)

(b) Find a subset of S that is a basis for W. (6%)

(c) Find an orthogonal basis for W. (5%)

◁注意背面尚有試題▶

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6) Let $T: R^3 \to R^3$ be a linear transformation given by

$$T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right)=\begin{bmatrix}0\\-1\\1\end{bmatrix}, T\left(\begin{bmatrix}0\\-1\\1\end{bmatrix}\right)=\begin{bmatrix}2\\1\\0\end{bmatrix}, T\left(\begin{bmatrix}1\\-1\\0\end{bmatrix}\right)=\begin{bmatrix}0\\0\\1\end{bmatrix}.$$

(a) Find a matrix A such that $T(\vec{x}) = A\vec{x}, \forall\ \vec{x} \in R^3$. (10%)

(b) Find the rank and nullity of T. (10%)

7) Let $A=\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}$.

(a) Find the eigenvalues of A. (8%)

(b) Find a diagonal matrix D similar to A. (4%)